Problems: Antennas and Free-Space Propagation
EL-GY 6023. Wireless Communications

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In all the problems below, unless specified otherwise, ϕ is the azimuth angle and θ is elevation angle.

1. *EM wave*: Suppose the E-field is,

$$\mathbf{E}(x,y,z,t) = E_0 \mathbf{e}_y \cos(2\pi f t - kx).$$

   (a) What is direction of motion?
   (b) If the average power flux is 10^{-8} mW/m^2, what is E_0? Assume the characteristic impedance is $\eta_0 = 377\Omega$.
   (c) If the frequency is $f = 1.5$ GHz, what is k? What are the units of k?

2. *EM polarization*: An EM plane wave has a power flux of 10^{-8} mW/m^2 in a horizontal polarization (along the x-axis) and $2(10)^{-8}$ mW/m^2 in a vertical polarization (along the y-axis). Assume the characteristic impedance is $\eta_0 = 377\Omega$.
   (a) What is combined E-field value and its direction?
   (b) What is the direction of motion?
   (c) What is the total power flux?

3. *dBm to linear conversions:*
   (a) Convert the following to mW: 17 dBm, -73 dBm, -97 dBW.
   (b) Convert the following to dBm: 250 mW, $8(10)^{-8}$ W, $5(10)^{-6}$ mW

4. *Spherical-cartesian conversions:*
   (a) Convert $(r,\phi,\theta) = (2, 45^\circ, 30^\circ)$ to (x,y,z).
   (b) Convert $(x,y,z) = (1,2,3)$ to (r,ϕ,θ).
   (c) Convert $(x,y,z) = (1,-2,3)$ to (r,ϕ,θ).

5. *Rotation matrices*: The *rotation matrix* $R(\theta,\phi)$ is the 3×3 matrix such that $R(\theta,\phi)\mathbf{r}$ rotates the vector $\mathbf{r}$ by an angle pair (θ,ϕ). You can read more about this on wikipedia or other sources.
   (a) Find the entries of $R(\theta,\phi)$.
   (b) Show $R(\theta,\phi)$ is *orthogonal* meaning $R(\theta,\phi)^{-1} = R(\theta,\phi)^{\mathsf{T}}$.

(c) Is

$$R(\theta_1, \phi_1)R(\theta_2, \phi_2) = R(\theta_2, \phi_2)R(\theta_1, \phi_1)?$$

That is, do the order of rotations matter? Prove or find a counter-example.

6. *Angular areas:* Find the angular area in steradians of following sets of angles:

   (a) $A_1 = \{(\phi, \theta) \mid \phi \in [-30, 30],\ \theta \in [-90, 90]\}$

   (b) $A_2 = \{(\phi, \theta) \mid \phi \in [-30, 30],\ \theta \in [-45, 45]\}$

7. *Directivity:* Suppose an antenna radiates power uniformly in the angular beam $\phi \in [-30, 30], \theta \in [-45, 45]$ and radiates no power at other angles. What is the directivity of the antenna?

8. *Radiation intensity:* A 170 cm x 40 cm object (roughly the size of a human) is 800m from a base station. If the antenna transmits 250 mW isotropically, how much power reaches the human? Use reasonable approximations that the human is far from the transmitter.

9. *Radiation integration:* The radiation density for some antenna is,

$$U(\phi, \theta) = A\cos^2(\phi), \quad A = 100\text{mW/sr}.$$

   (a) What is the total radiated power in mW and dBm?

   (b) What is the directivity of the antenna in linear scale and in dBi?

10. *Numerically integrating patterns:* Write a short MATLAB function,

```
function [totPow, dir] = powerDirectivity(az,el,E,eta)
```

that computes the total power and directivity as a function of the E-field values. The E field is specified as a complex matrix `E(i,j)` at angles `az(i),el(j)`. You can assume that the angles are uniformly spaced over the total angular space. The total power output `totPow` should be a scalar representing the total radiated power in dBm and the directivity output should be `dir(i,j)` should be a matrix.

11. *Friis' Law*: A transmitter radiates 100 mW at a carrier $f_c = 2.1$ GHz with a directional gain of $G_t = 10$ dBi. Suppose the receiver is $d = 200$ m from the transmitter and the path is free space. What is the received power if:

   (a) The effective received aperture is 1 cm^2.

   (b) The receiver gain is $G_r = 5$ dBi.