Multiple Antennas and Beamforming

EL-GY 6023. WIRELESS COMMUNICATIONS

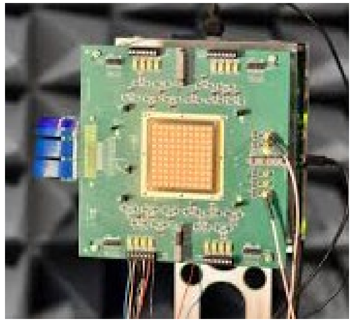
PROF. SUNDEEP RANGAN

Outline

- Antenna arrays and the Spatial Signature
- Receive Beamforming and SNR Gain
- Array Factor
- Multiple paths and Diversity
- Transmit Beamforming

Antenna Arrays

- **Antenna arrays**: Structure with multiple antennas
  - At TX and/or RX
  - Key to 5G mmWave and massive MIMO
- Two key benefits
- **Beamforming**: This lecture
  - Concentrate power in particular directions
  - Increases SNR and may enable spatial diversity
  - Requires arrays at *either* TX or RX
- **Spatial multiplexing**: Next lecture
  - Enables transmission in multiple virtual paths
  - Increases degrees of freedom
  - Requires multiple antennas at *both* TX and RX



IBM 28 GHz array
32 element dual polarized array
Sadhu et al, ISSCC 2017

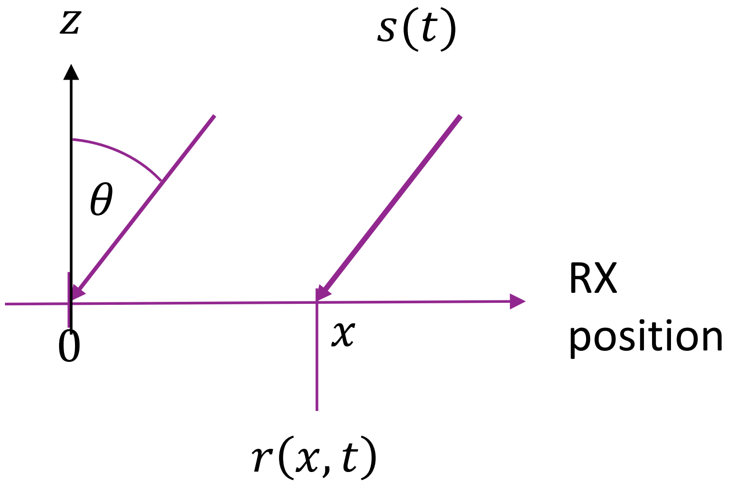
Aurora C-Band Massive MIMO array
64 elements, 5-6 GHz
https://www.taoglas.com/

Multiple Receive Antennas

- Single Input Multiple Output
  - One TX antenna
  - M RX antennas
- Transmit a scalar signal $x(t)$
- Receive a vector of signals:
  - $\boldsymbol{r}(t) = \left(r_1(t), \dots, r_M(t)\right)^T$
- What is the channel from $x(t)$ to $\boldsymbol{r}(t)$?
- Want channel in complex baseband

Channel vs. Position

- To understand SIMO channel, consider single path channel
  - AoA of θ relative to z-axis
  - Delay τ_0 to origin
  - Gain A is constant close to origin
- Transmit signal $s(t)$ and look at response at position x
- Consider a RX position close to origin
  - $\mathrm{B}|x| \ll f_c\lambda,\ B =$ bandwidth of $s(t)$
- Phase rotation with displacement:
  - Baseband response at x is (proof on next slide):



$$r(x,t) \approx \underbrace{e^{2\pi j x \sin\theta/\lambda}}_{\text{Phase rotation with } x} \underbrace{r(0,t)}_{\text{Response at } x\,=\,0}$$

Proof of Phase Rotation with Displacement

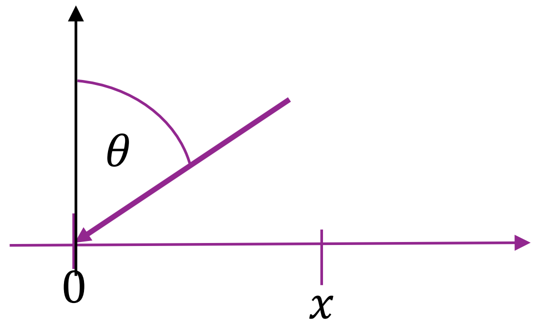
- Delay of path at x is: $\tau(x) = \tau_0 - \frac{\mathrm{x}\sin\theta}{c}$
- Baseband response at x:

$$r(x,t) = Ae^{-j\omega_c\tau_0}e^{2\pi jx\sin\theta/\lambda}s(t-\tau(x))$$

- Also, $s\big(t-\tau(x)\big) \approx s(t-\tau_0)$ if $\mathrm{B}|\tau(x)-\tau_0| \ll 1$
- But, by assumption of small displacement:

$$\mathrm{B}|\tau(x)-\tau_0| \le \frac{B|x|}{c} = \frac{B|x|}{\lambda f_c} \ll 1$$

- Hence, $r(x,t) \approx Ae^{-j\omega_c\tau_0}e^{2\pi jx\sin\theta/\lambda}s(t-\tau_0) = e^{2\pi jx\sin\theta/\lambda}r(0,t)$



RX position

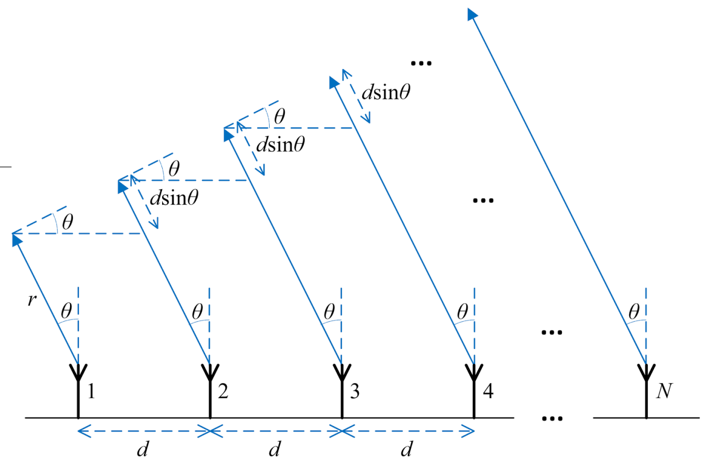
Response for a ULA

- Uniform Linear array (ULA)
  - M antenna positions spaced d apart
- Transmit signal $s(t)$
  - Channel single path with AoA θ, gain A
- Response at position: $r_m(t) = Ae^{-j\omega\tau_0} e^{2\pi j(n-1)d\sin\ \theta/\lambda} s(t-\tau_0)$
- SIMO frequency response is:

$$\boldsymbol{h}(\theta,\omega) = Ae^{-j\omega\tau_0} \begin{bmatrix} e^{2\pi j 0 d\sin\theta/\lambda} \\ \vdots \\ e^{2\pi j(M-1)d\sin\theta/\lambda} \end{bmatrix} = g(\omega)\boldsymbol{u}(\theta)$$

Scalar response at $x = 0$

Phase shifts across elements

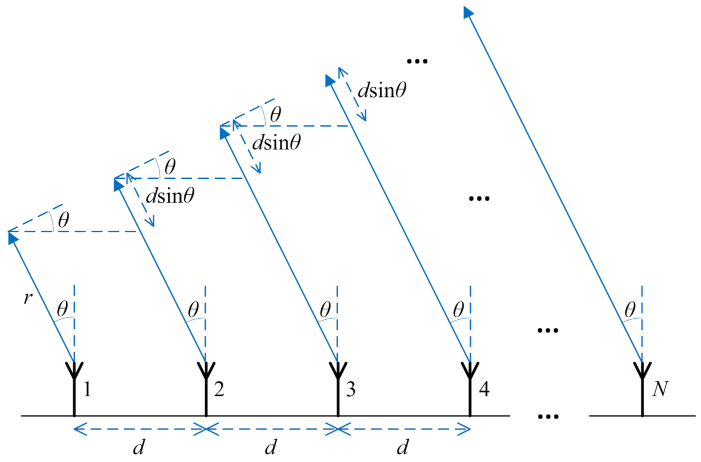


Response Decomposition

- For a single path channel, the frequency response has two components:

$$\boldsymbol{h}(\theta,\omega) = g(\omega)\boldsymbol{u}(\theta)$$

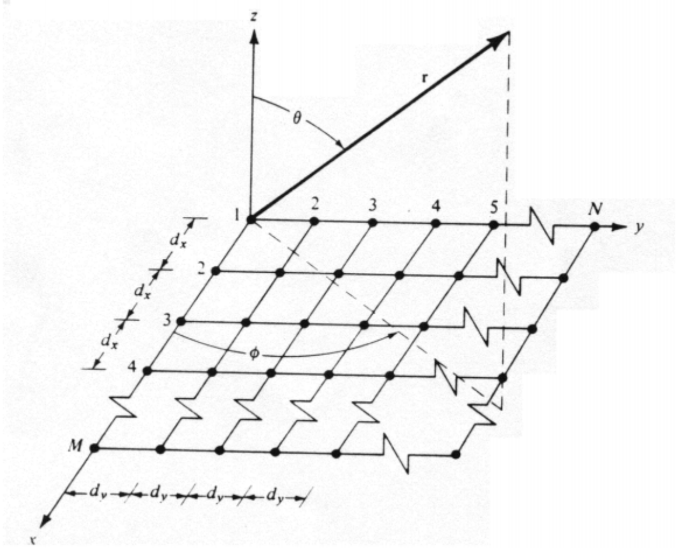
- Scalar channel response, $g(\omega)$
  - $g(\omega) = Ae^{-j\omega\tau_0}$
  - Response at a reference position in array



- Vector spatial signature, $\boldsymbol{u}(\theta)$
  - $\boldsymbol{u}(\theta) = \begin{bmatrix} e^{2\pi j 0 d \sin\theta/\lambda} \\ \vdots \\ e^{2\pi j (M-1) d \sin\theta/\lambda} \end{bmatrix}$
  - Vector of phase shifts from the reference
  - Also called the steering vector (reason for name will be clear later)

Array Response in 3D

- Many arrays place elements over 2D area
- Uniform rectangular array (URA):
  - $M \times N$ grid of elements
  - Spaced d_x and d_y
  - Also called uniform planar array (UPA)
- Incident angle $\Omega = (\phi, \theta)$
  - (Azimuth, elevation) or (azimuth, inclination)
- Spatial signature:
  - $u_{mn}(\Omega) =$ complex response to antenna (m, n)
  - $u_{mn}(\Omega) = \exp\left[\frac{2\pi i}{\lambda}\left(md_x \sin\theta \cos\phi + nd_y \sin\theta \sin\phi\right)\right]$



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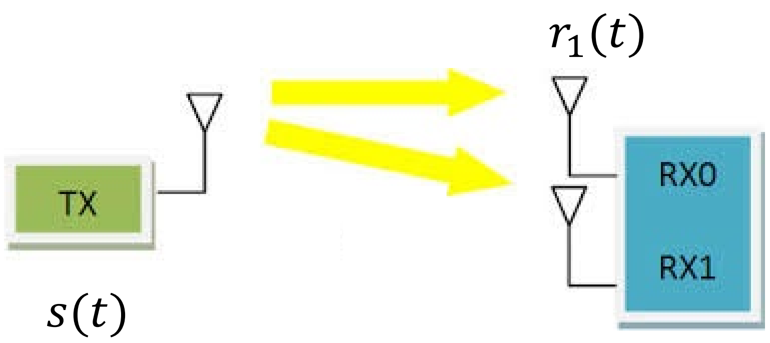
- ❑Receive Beamforming and SNR Gain
- ❑Array Factor
- ❑Multiple paths and Diversity
- ❑Transmit Beamforming

Multiple Receive Antennas

- Single Input Multiple Output
  - One TX antenna
  - M RX antennas



- Transmit a scalar signal $s(t)$
- Receive a vector of signals:
  - $\boldsymbol{r}(t) = \left(r_1(t), \ldots, r_M(t)\right)^T$

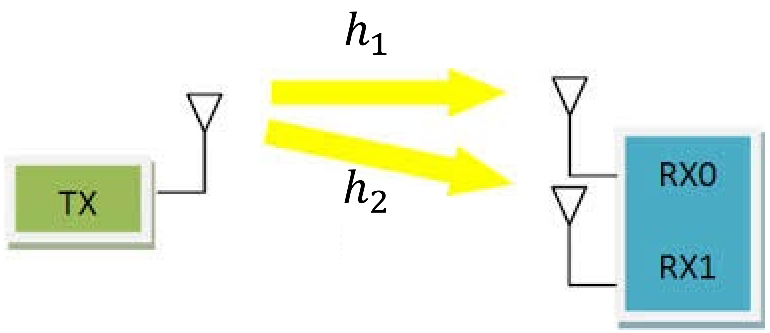
- Basic question: How do we decode signal $x(t)$ from vector $\boldsymbol{r}(t)$?

Scalar Multiple Channel Problem

- Consider transmission of a single symbol x
- Receive a vector across M *channels*:

$$\boldsymbol{r} = \boldsymbol{h}x + \boldsymbol{n} = \begin{pmatrix} h_1 \\ \vdots \\ h_M \end{pmatrix} x + \begin{pmatrix} n_1 \\ \vdots \\ n_M \end{pmatrix}$$

  - x: Scalar TX symbol
  - $\boldsymbol{h}$: Vector of channel weights, $\boldsymbol{n}$: Vector of noise
- Channel can be from many different paths:
  - multiple times, frequencies or antennas
- Applies to a single degree of freedom (time or frequency)
- Question: How do we detect scalar x from vector $\boldsymbol{r}$?

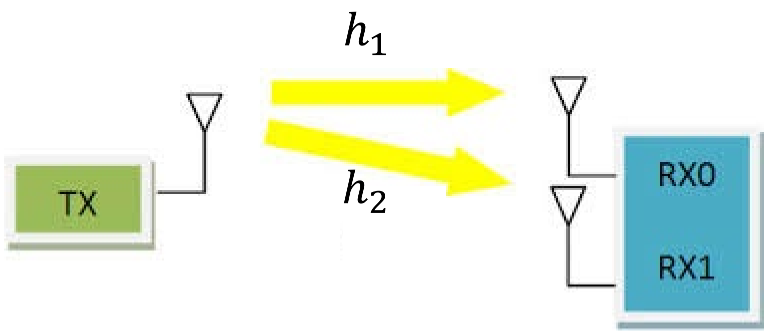


Linear Combining

- RX model: $\boldsymbol{r} = \boldsymbol{h}x + \boldsymbol{n}$
  - 1 input, M outputs
- Linear combining: Take a linear combination

$$z = \boldsymbol{w}^*\boldsymbol{r} = (\boldsymbol{w}^*\boldsymbol{h})x + \boldsymbol{w}^*\boldsymbol{n}$$
$$= \alpha x + v$$

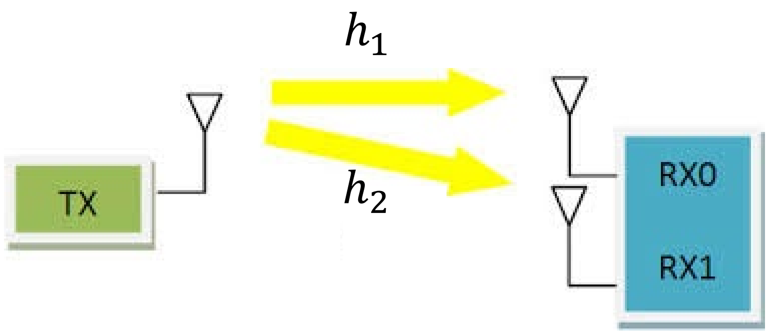
- $\boldsymbol{w}$ is called the weighting vector
  - Called the beamforming vector for multiple antennas
- Creates effective SISO channel:
  - 1 input x, 1 output symbol z
  - Gain: $\alpha = \boldsymbol{w}^*\boldsymbol{h}$
  - Noise: $v = \boldsymbol{w}^*\boldsymbol{n}$



Linear Combining Analysis

- Linear combining: $z = \boldsymbol{w}^*\boldsymbol{r} = (\boldsymbol{w}^*\boldsymbol{h})x + \boldsymbol{w}^*\boldsymbol{n}$
  - Gain: $\alpha = \boldsymbol{w}^*\boldsymbol{h}$
  - Noise: $v = \boldsymbol{w}^*\boldsymbol{n}$
- Analysis: Let
  - $E_x = E|x|^2$ = average symbol energy
  - Assume noise $n_m \sim CN(0, N_0)$ (i.i.d. complex Gaussian noise)
- Then, after combining;
  - Signal energy = $|\boldsymbol{w}^*\boldsymbol{h}|^2 E_x$
  - Noise: v is Gaussian with $E|v|^2 = \|\boldsymbol{w}\|^2 N_0$
  - SNR is:

$$\gamma = \frac{|\boldsymbol{w}^*\boldsymbol{h}|^2 E_x}{\|\boldsymbol{w}\|^2 N_0}$$

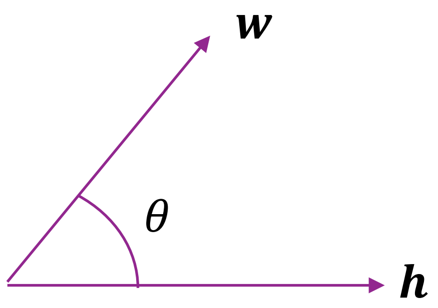


Maximum Ratio Combining

- From previous slide: SNR is $\gamma = \frac{|\boldsymbol{w}^*\boldsymbol{h}|^2 E_x}{\|\boldsymbol{w}\|^2 N_0}$
- Maximum ratio combining: Select BF vector to maximize SNR: $\widehat{\boldsymbol{w}} = \arg\max_{\boldsymbol{w}} \frac{|\boldsymbol{w}^*\boldsymbol{h}|^2 E_x}{\|\boldsymbol{w}\|^2 N_0}$
- Theorem: The MRC weighting vector and maximum SNR is:

$$\boxed{\widehat{\boldsymbol{w}} = c\boldsymbol{h} \Rightarrow \gamma_{MRC} = \|\boldsymbol{h}\|^2 \frac{E_x}{N_0}}$$

  - Any constant $c \neq 0$ can be used. Constant does not matter
  - Align BF vector with the channel.



- Proof:
  - From Cauchy-Schwartz: $|\boldsymbol{w}^*\boldsymbol{h}|^2 = \|\boldsymbol{w}\|^2\|\boldsymbol{h}\|^2 \cos\theta$
  - Hence, $\gamma = \|\boldsymbol{h}\|^2 \frac{E_x}{N_0} \cos\theta$
  - Maximized with $\cos\theta = 1 \Rightarrow \theta = 0$

MRC Gain

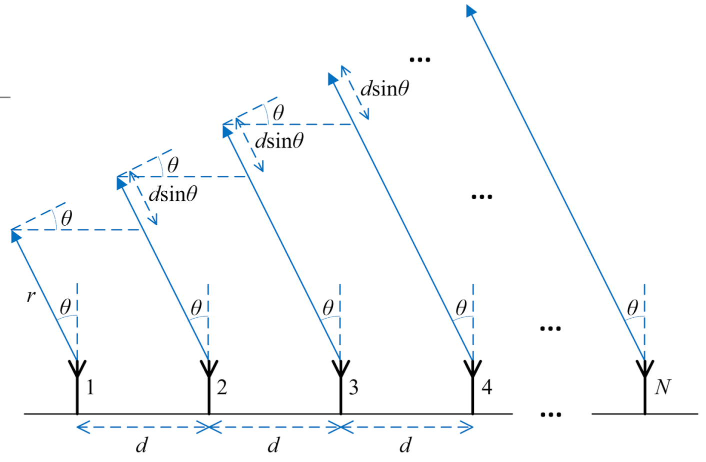
- SNR with MRC: $\gamma_{MRC} = \|\boldsymbol{h}\|^2 \frac{E_x}{N_0}$
- SNR on channel i is: $\gamma_i = \frac{|h_i|^2 E_x}{N_0}$
- Average SNR is: $\gamma_{avg} = \frac{1}{M}\sum_{i=1}^{M} \gamma_i = \frac{1}{M}\sum_{i=1}^{M} |h_i|^2 \frac{E_x}{N_0} = \frac{1}{M} \|\boldsymbol{h}\|^2 \frac{E_x}{N_0}$
- MRC increases SNR by a factor of M relative to average per channel SNR
- Beamforming gain = $\frac{\gamma_{MRC}}{\gamma_{avg}} = M$
- Example: Suppose average SNR per antenna is 10 dB.
  - With $M = 16$ antennas and MRC, SNR $= 10 + 10\log_{10}(16) = 10 + 4(3) = 22$ dB
  - Gain increases significantly!

RX Beamforming

- Recall model for a single path channel:

$$\boldsymbol{r} = g_0 \boldsymbol{u}(\Omega) x + \boldsymbol{n}$$

  - $\boldsymbol{u}(\Omega)$ = spatial signature on that angle, Ω= angle of arrival
  - g_0 = gain at reference position in array
  - x = transmitted symbol



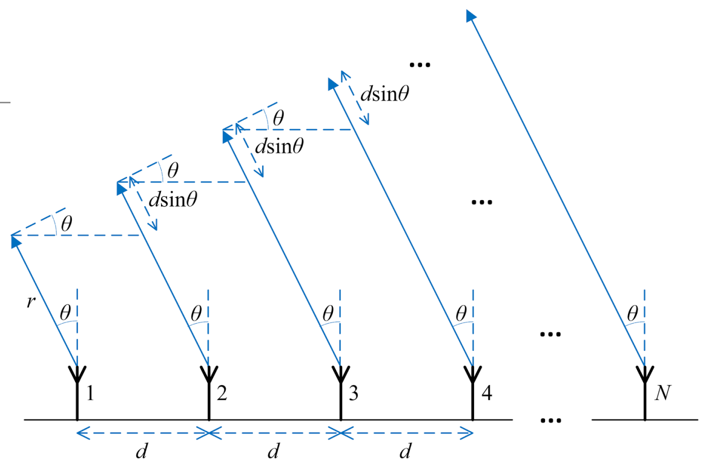
- RX beamforming is just linear combining across antennas

$$z = \boldsymbol{w}^* \boldsymbol{r}$$

  - $\boldsymbol{w}$ is called the beamforming vector
  - By convention, we assume $\|\boldsymbol{w}\| = 1$
  - Geometric interpretation to be given shortly

MRC Beamforming

- Single path channel: $\boldsymbol{r} = g_0 \boldsymbol{u}(\Omega) x + \boldsymbol{n}$
- RX beamforming: $z = \boldsymbol{w}^* \boldsymbol{r}$
- SNR per antenna (before beamforming):
  - $\gamma_0 = \frac{E_x |g_0|^2}{N_0} |u_m(\Omega)|^2 = \frac{E_x |g_0|^2}{N_0}$
  - Assume $u_m(\Omega)$ includes only phase shifts
- SNR after BF: $\gamma = \frac{|\boldsymbol{w}^* \boldsymbol{u}(\Omega)|^2}{\|\boldsymbol{w}\|^2} \gamma_0$
- MRC beamforming: $\widehat{\boldsymbol{w}} = c\boldsymbol{u}(\Omega)$ and $\gamma = \|\boldsymbol{u}(\Omega)\|^2 \gamma_0 = M\gamma_0$
- Conclusions:
  - Optimal (MRC) beamforming vector is aligned to the spatial signature
  - Optimal SNR gain $= M$
  - Linear gain with number of antennas



Example Problem

- Consider a system
  - TX power = 23 dBm with antenna directivity = 10 dBi
  - Free space path loss $d = 1000$ m
  - Sample rate = 400 Msym/s
  - Noise energy = -170 dBm/Hz (including NF)
  - RX antenna directivity = 5 dBi and 8 elements
- Find SNR per antenna and SNR with MRC
- Solution: We get a 9 dB gain!

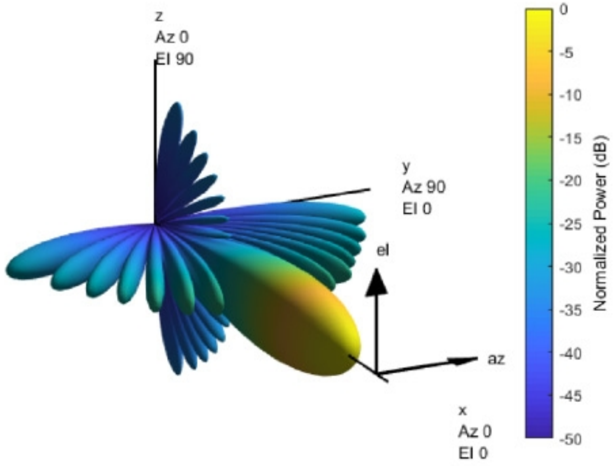
```
SNR per  ant:      0.59
SNR with MRC:      9.62
```

```
% SNR per antenna
plomni = fspl(dist, lambda);
EsN0Ant = ptx - plomni - 10*log10(bw) - Enoise + dirtx + dirrx;

% SNR with MRC
EsN0MRC = EsN0 + 10*log10(nantrx);
```

MATLAB Phased Array Toolbox

- Powerful toolbox
- Routines for:
  - Defining and visualizing arrays
  - Computing beam patterns
  - Beamforming
  - MIMO
  - Radar
  - …

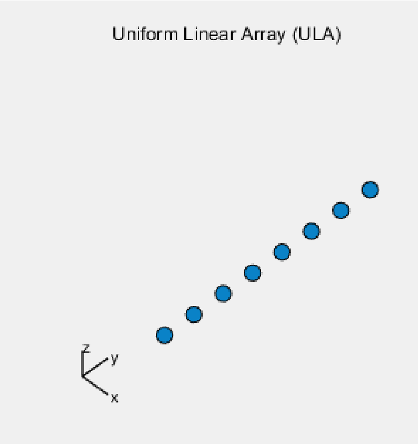


Example: Defining a ULA

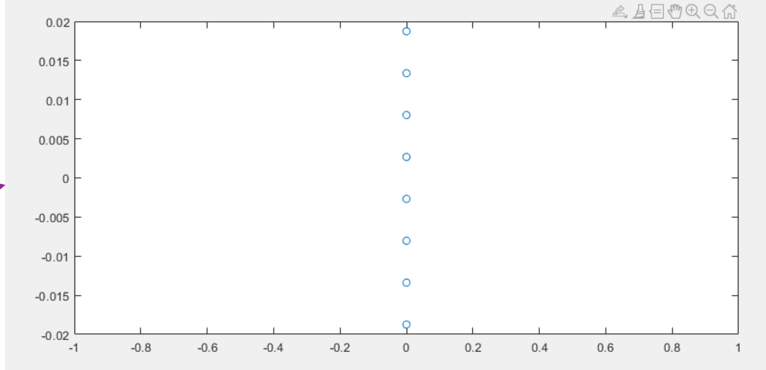
- ❑ Define and view the array
- ❑ Can display array:
  - Using viewArray command
  - Or, manually

```
%% Uniform Linear Array
% We first define a simple uniform linear array
fc = 28e9;              % frequency
lambda = physconst('LightSpeed')/fc;
dsep = 0.5*lambda;      % element spacing
nant = 8;               % Number of elements
arr = phased.ULA(nant,dsep);

% View the array
viewArray(ula,'Title','Uniform Linear Array (ULA)')
```




```
elemPos = arr.getElementPosition();
clf('reset');
plot(elemPos(1,:), elemPos(2,:), 'o');
```



Computing the Spatial Signature

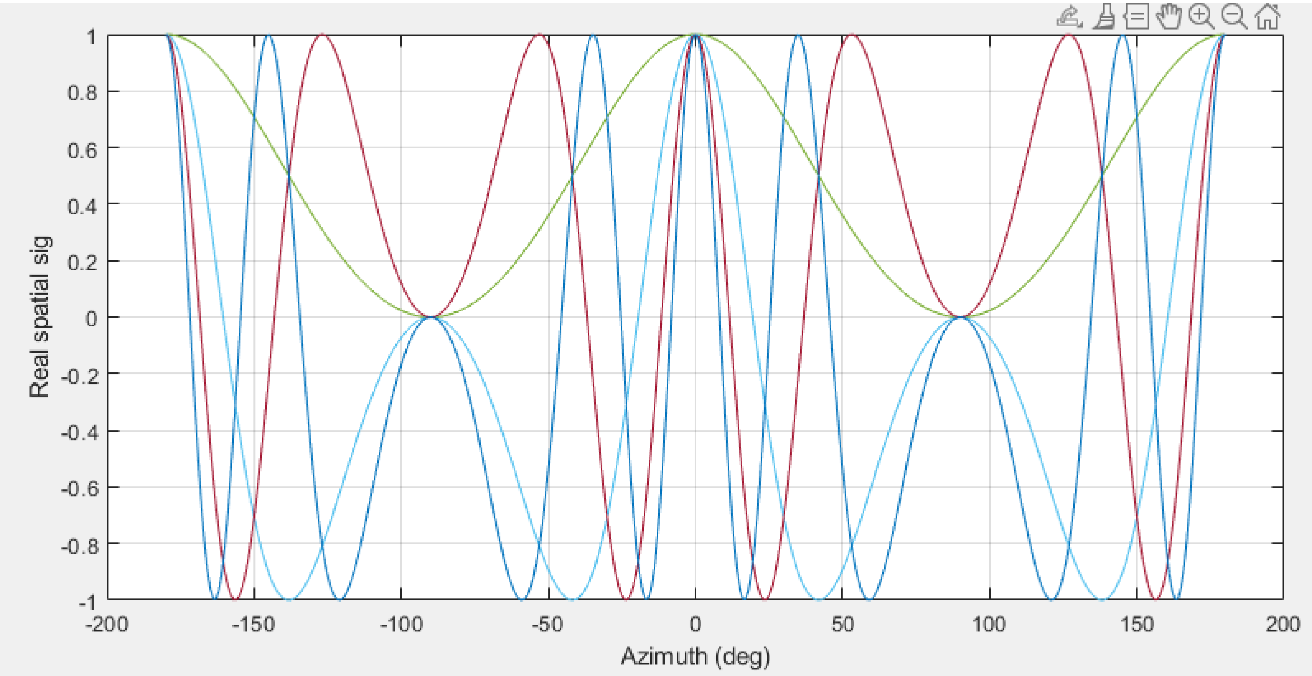
- Compute the spatial signature with the SteeringVector object

```
% Create a steering vector object
sv = phased.SteeringVector('SensorArray',arr);

% Angles to compute the SVs
npts = 361;
az = linspace(-180,180,npts);
el = zeros(1,npts);
ang = [az; el];

% Matrix of steering vectors
% This is an nant x npts matrix in this case
u = sv(fc, ang);

% Plot of the real components
plot(az, real(u)');
grid on;
xlabel('Azimuth (deg)')
ylabel('Real spatial sig');
```

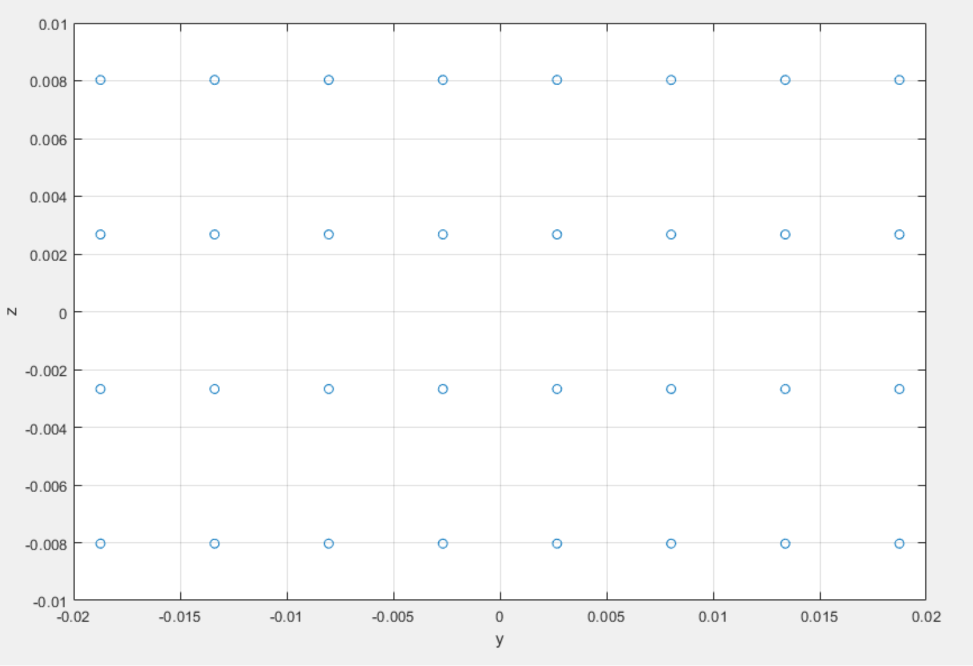


Example: Defining a URA

- Define and view the array
- Use the phased.URA class
- Can compute steering vector similarly

```
% Construct the array
nant = [4,8];
dsep = 0.5*lambda;
arr = phased.URA(nant,dsep,'ArrayNormal','x');

% Plot the array.
% You can also use, arr.viewArray()
elemPos = arr.getElementPosition();
clf('reset');
plot(elemPos(2,:), elemPos(3,:), 'o');
grid on;
xlabel('y');
ylabel('z');
```



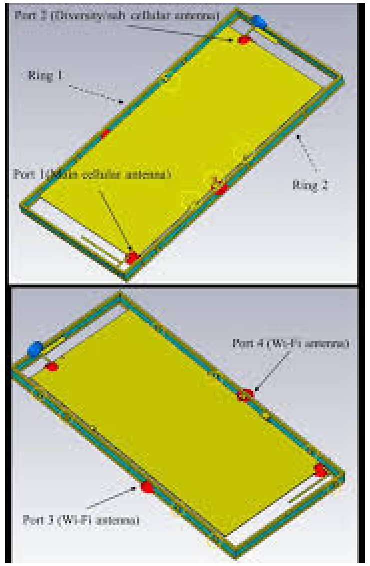
A 4 x 8 URA with normal axis aligned on x

Multiple Antennas in Commercial Systems

- Sub 6 GHz systems: Mostly 1 to 4 antennas on base stations or smart phones
- Form factor restricts larger number of antennas

WiFi Router
Linksys AC2200 with 4TX/RX

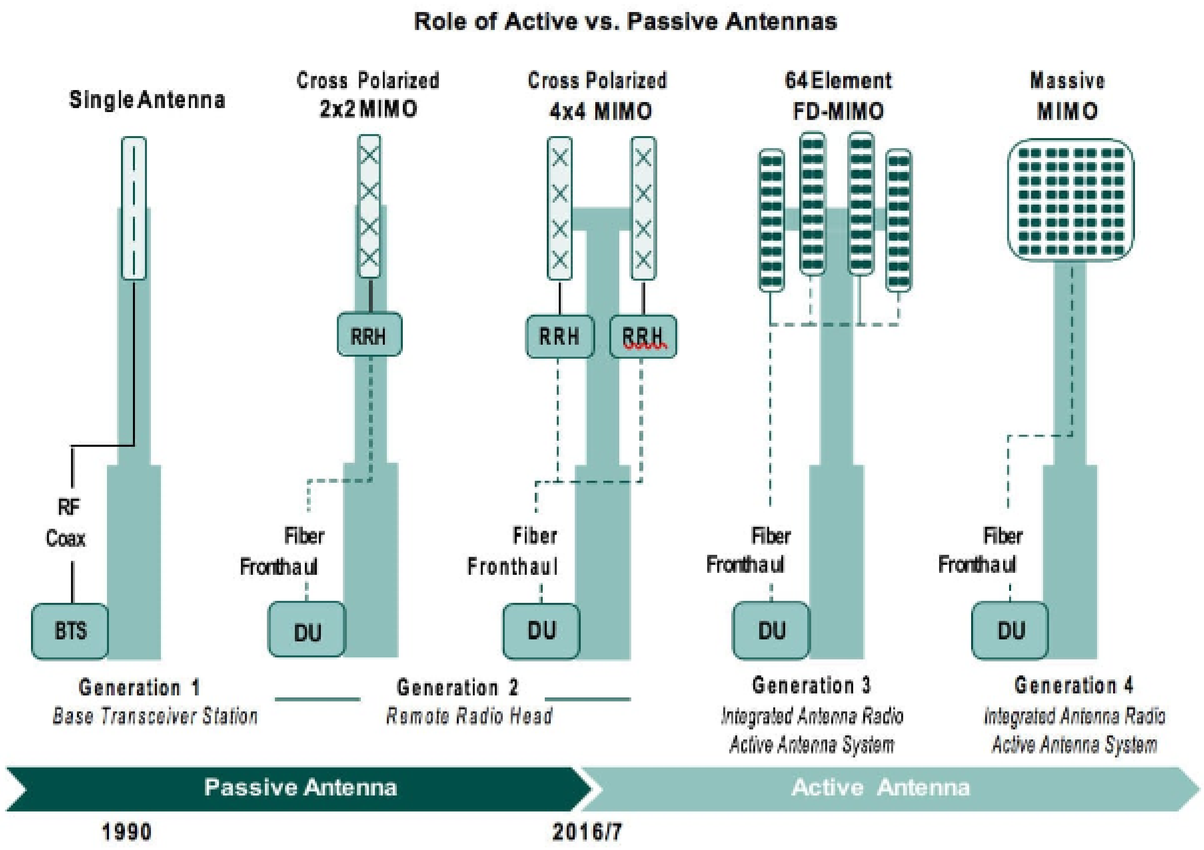
2x2 LTE base station antenna
Cros-polarization
16 dBi element gain, 90 deg sector
750x120x60mm



K. Zhao, S. Zhang, K. Ishimiya, Z. Ying and S. He, "Body-Insensitive Multimode MIMO Terminal Antenna of Double-Ring Structure," in *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 5, pp. 1925-1936, May 2015.

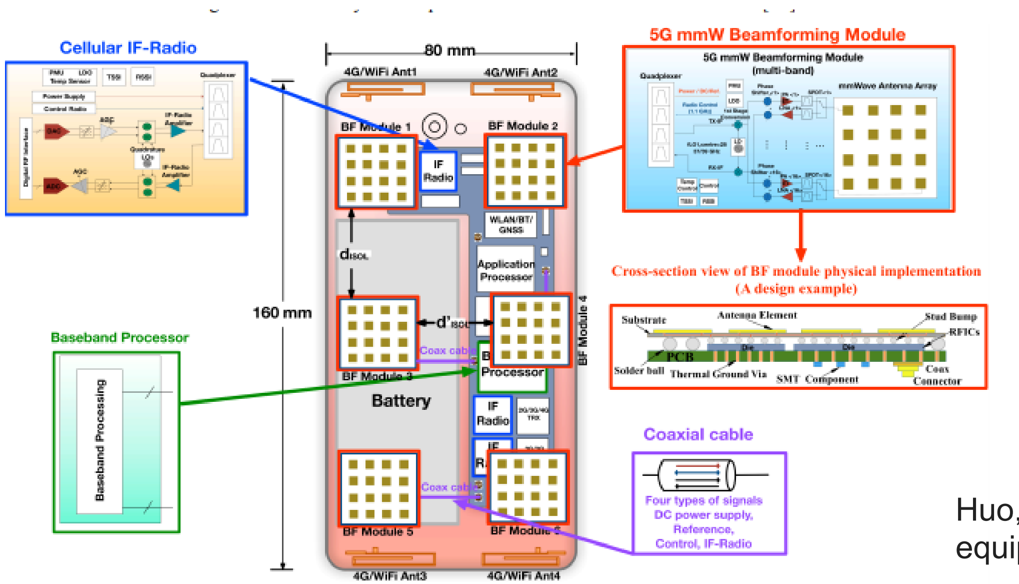
Massive MIMO

- Massive MIMO:
  - Many base station antennas
  - 64 to 128 in many systems today
- Significant capacity increase
  - Typically 8x by most estimates
- Use SDMA
  - Will discuss this later

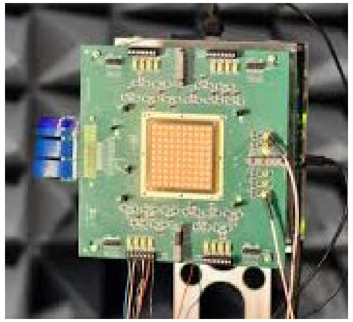


Beamforming and MmWave

- To compensate for high isotropic path loss, mmWave systems need large number of antennas
- 5G handsets: Multiple arrays with 4 to 8 antennas each
- 5G base stations: 64 to 256 elements



Huo, Yiming, et al. "Cellular and WiFi co-design for 5G user equipment." *2018 IEEE 5G World Forum (5GWF)*. IEEE, 2018.



IBM 28 GHz array
32 element dual polarized array
Sadhu et al, ISSCC 2017

In-Class Problem: Simple QPSK simulation

- Simulate QPSK transmission over a single path channel

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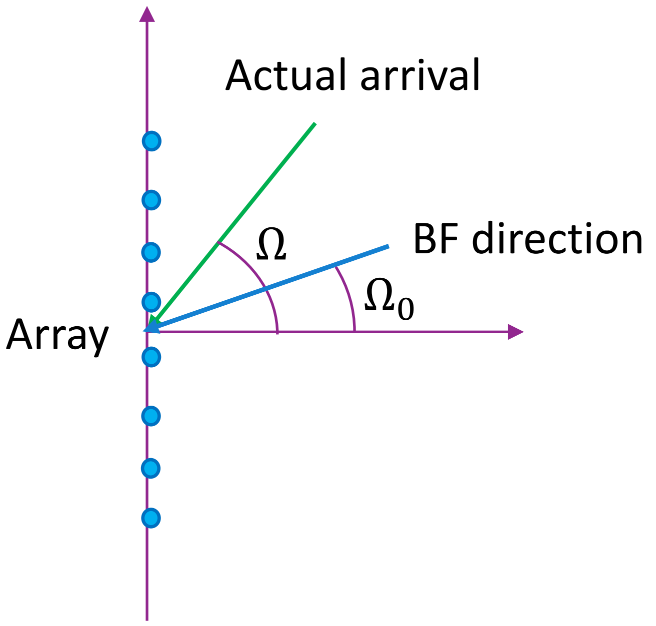
- ❑ Receive Beamforming and SNR Gain
- ❑ Array Factor
- ❑ Multiple Paths and Diversity
- ❑ Transmit Beamforming

Array Factor

- Suppose RX aligns antenna for AoA $\Omega_0 = (\theta_0, \phi_0)$
- But, signal arrives from AoA $\Omega = (\theta, \phi)$
- Define the (complex) array factor

$$AF(\Omega, \Omega_0) = \widehat{\boldsymbol{w}}^*(\Omega_0)\boldsymbol{u}(\Omega) = \frac{1}{\sqrt{M}}\boldsymbol{u}^*(\Omega_0)\boldsymbol{u}(\Omega)$$

  - Assume $||\widehat{w}|| = 1$
  - Indicates directional gain as a function of AoA θ
  - Dependence on θ_0 often omitted



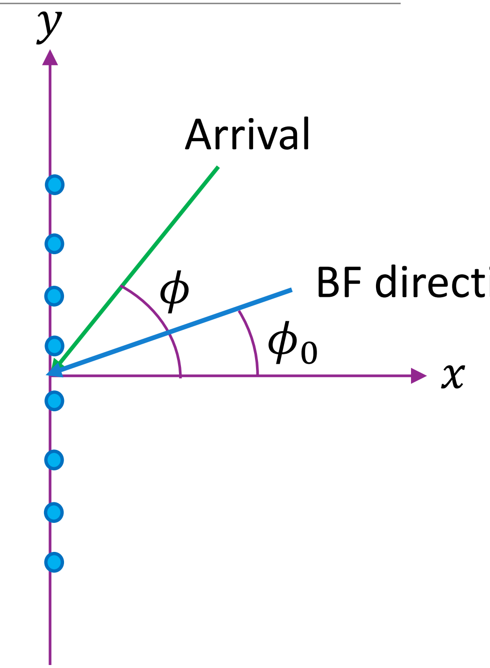
- SNR gain = $|AF(\Omega, \Omega_0)|^2$
  - Max value = M
  - Usually measured in dBi (dB relative to isotropic)
  - Also called the array response

Uniform Linear Array

- Spatial signature (for azimuth angle ϕ):
  - $\boldsymbol{u}(\phi) = \left[1, e^{j\beta}, \dots, e^{i(M-1)\beta}\right]^T, \ \beta = \frac{2\pi d \cos\phi}{\lambda}$
  - Note change from $\sin\theta$ to $\cos\phi$. (Array aligned on y-axis)
- Optimal BF vector for AoA ϕ_0
  - $\widehat{\boldsymbol{w}}(\phi_0) = \frac{1}{\sqrt{M}} \boldsymbol{u}(\phi_0)$ (Note normalization)
- Array factor:

$$AF(\phi, \phi_0) = \widehat{\boldsymbol{w}}(\phi_0)^* \boldsymbol{u}(\phi) = \frac{e^{j(M-1)\gamma/2}}{\sqrt{M}} \frac{\sin(M\gamma/2)}{\sin(\gamma/2)},$$

  - $\gamma = \frac{2\pi d}{\lambda}(\cos\phi - \cos\phi_0),$
- Antenna gain: $|AF|^2 = \frac{\sin^2(M\gamma/2)}{M\sin^2(\gamma/2)}$

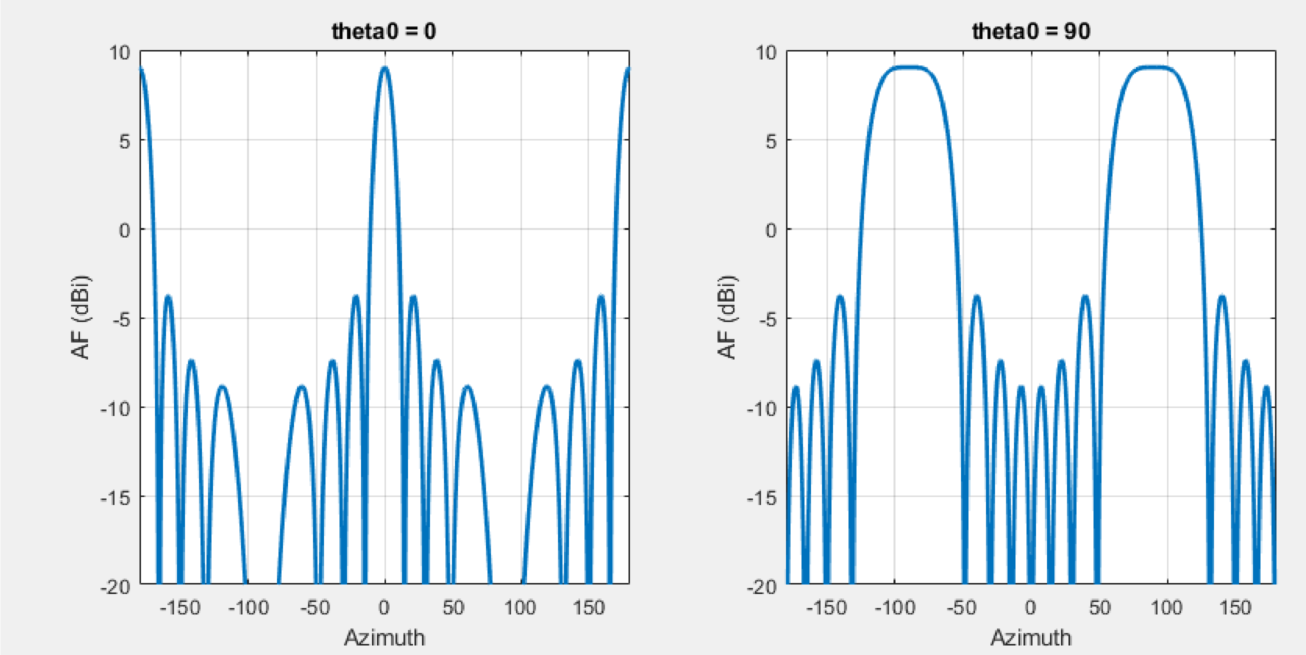


Antenna Gain for ULA

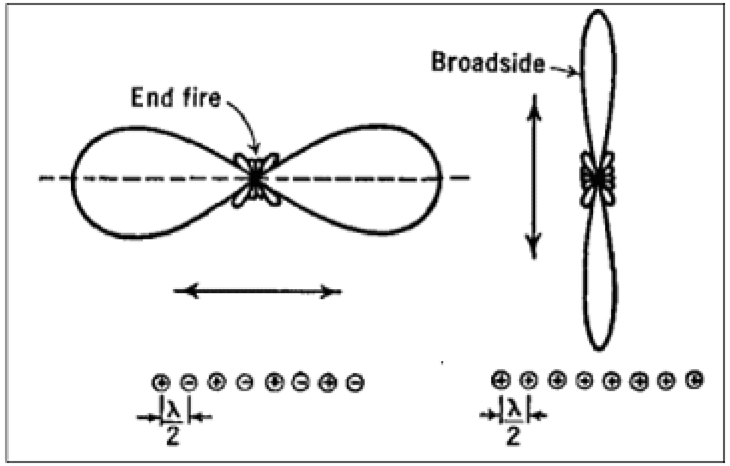
$d = \lambda/2\,, \qquad M = 8$

Broadside: $\theta_0 = 0$

Endfire: $\theta_0 = 90$



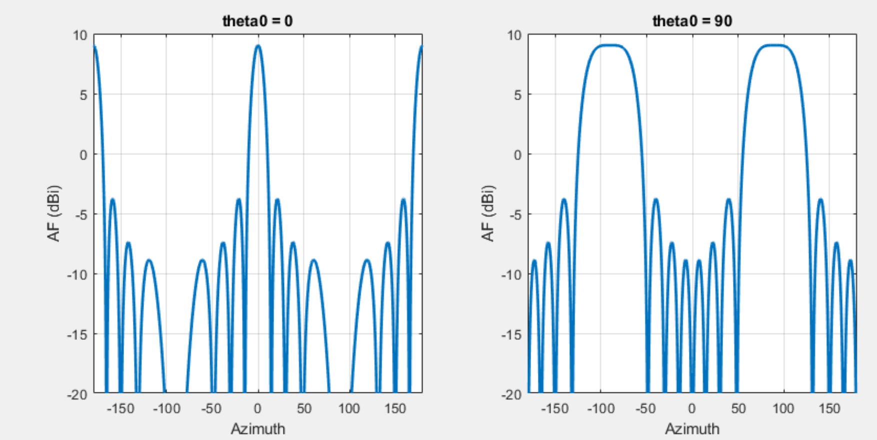
- Maximum gain of
- Note:
  - Endfire vs. broadside
  - Beamwidth $\propto\ 1/M$



Plotting the Array Factor

- ❑Create a SteeringVector object
- ❑Get steering vectors
- ❑Compute inner products

```
% Create a steering vector object
sv = phased.SteeringVector('SensorArray',arr);

% Reference angles to plot the AF
azPlot = [0, 90];
nplot = length(azPlot);
```

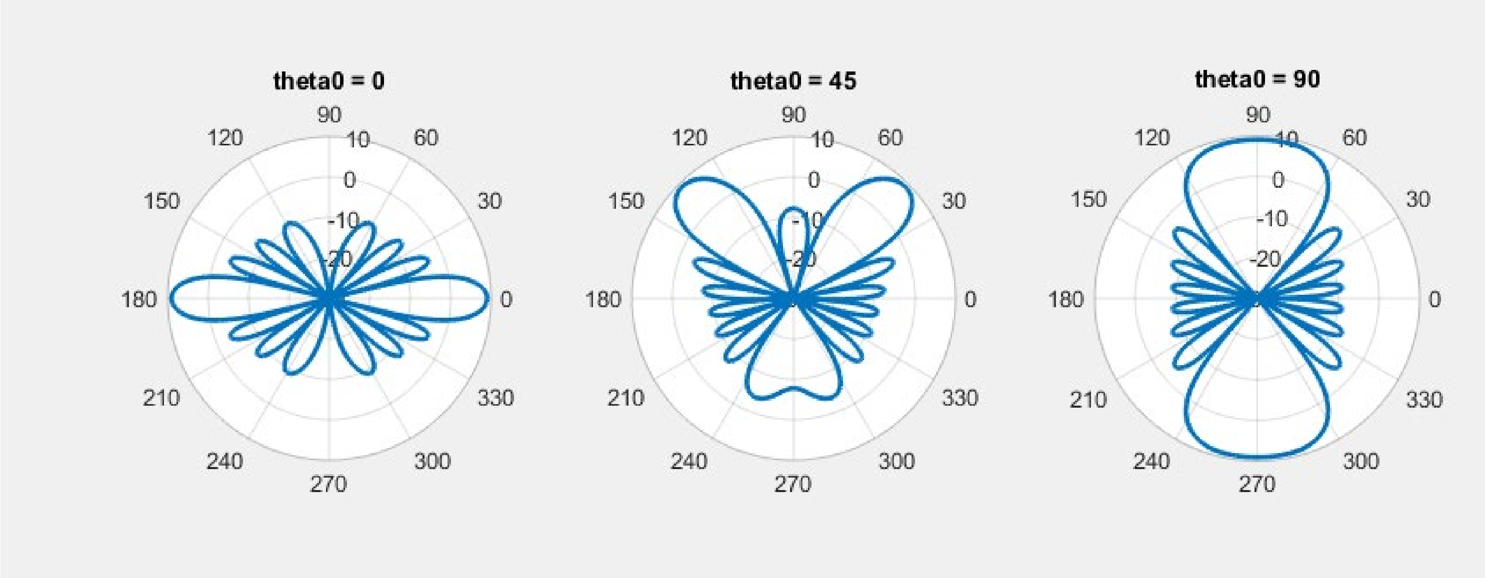



```
for iplot = 1:nplot
    % Get the SV for the beam direction.
    % Note:  You must call release method of the sv
    % before each call since it expects the same size
    % of the input
    ang0 = [azPlot(iplot); 0];
    sv.release();
    u0 = sv(fc, ang0);

    % Normalize the direction
    u0 = u0 / norm(u0);

    % Get the SV for the AoAs.  Take el=0
    npts = 1000;
    az = linspace(-180,180,npts);
    el = zeros(1,npts);
    ang = [az; el];
    sv.release();
    u = sv(fc, ang);

    % Compute the AF and plot it
    AF = 10*log10( abs(sum(conj(u0).*u, 1)).^2 );

    % Plot it
    subplot(1,nplot,iplot);
    plot(ang(1,:), AF, 'LineWidth', 2);
end
```

Polar Plot

- ❑ Useful to visualize in polar plot
- ❑ Note key features:
  - Direction of maximum gain
  - Sidelobes
  - Pattern repeated on reverse side

```
% Polar plot
AFmin = -30;
subplot(1,nplot,iplot);
polarplot(deg2rad(az), max(AF, AFmin),'LineWidth', 2);
rlim([AFmin, 10]);
grid on;
```



Key Statistics

Full null beamwidth (zero to zero)

Half power beamwidth (-3dB to -3dB)

First sidelobe level

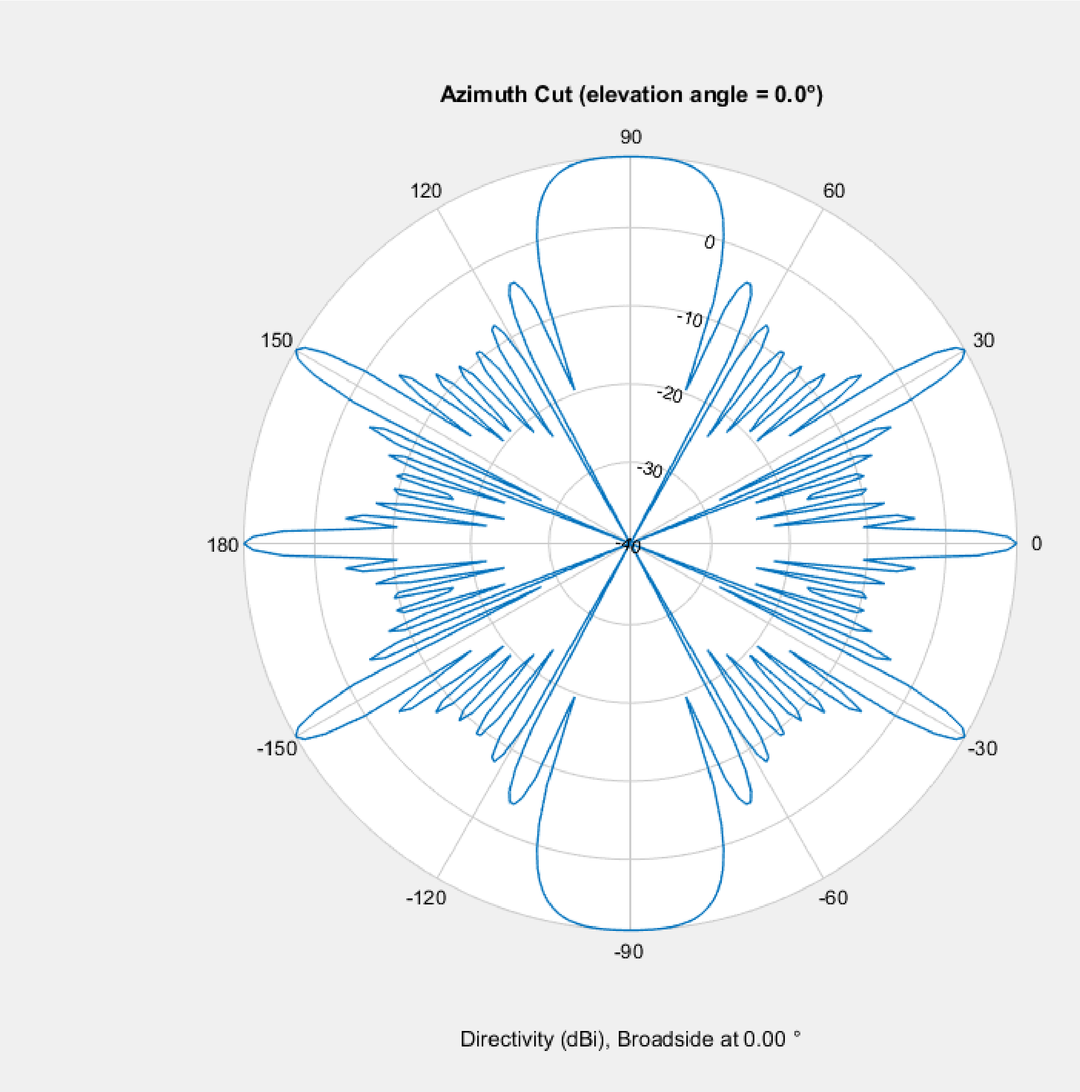
	Broadside ($\theta_0 = \pi/2$)	End-fire ($\theta_0 = 0$)
FNBW	$2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{N\Delta}\right)\right]$ (30°)	$2\cos^{-1}\left(1 - \frac{\lambda}{N\Delta}\right)$ (83°)
HPBW	$2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.39\lambda}{\pi N\Delta}\right)\right]$ (13°)	$2\cos^{-1}\left(1 - \frac{1.39\lambda}{\pi N\Delta}\right)$ (54°)
FSLL	$\frac{1}{N\left\lvert\sin\left(\frac{3\pi}{2N}\right)\right\rvert}$ (-13 dB)	$\frac{1}{N\left\lvert\sin\left(\frac{3\pi}{2N}\right)\right\rvert}$ (-13 dB)
D_0	$2N\Delta/\lambda$ (9 dB)	$4N\Delta/\lambda$ (12 dB)

- From Jacobs University slides
- Values in () for: $d = \lambda/2\,,\ M = 8$

Grating Lobes

- When $d > \frac{\lambda}{2}$
- Obtain multiple peaks
- Does not direct gain in one direction

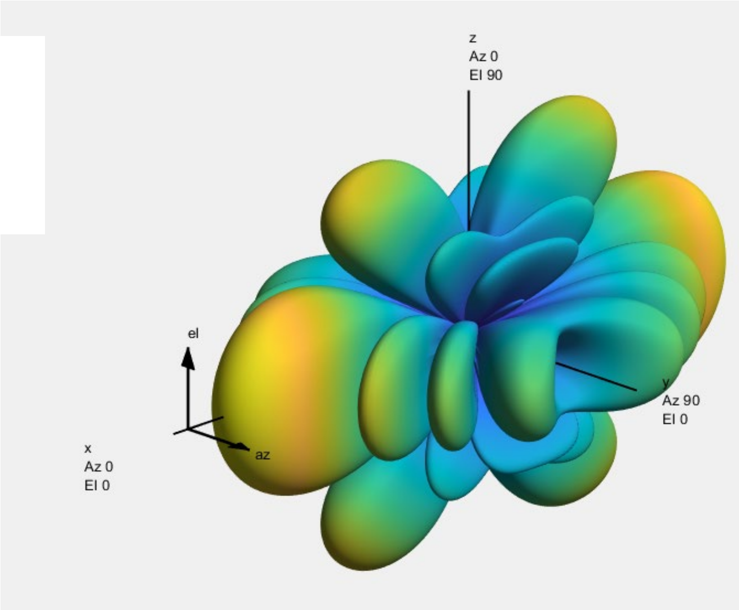
```
dsep = 2*lambda;       % element spacing
nant = 8;              % Number of elements
arr = phased.ULA(nant,dsep);

% Get the SV for the beam direction.
ang0 = [0; 0];
sv.release();
u0 = sv(fc, ang0);

arr.patternAzimuth(fc,'Weights', u0);
```



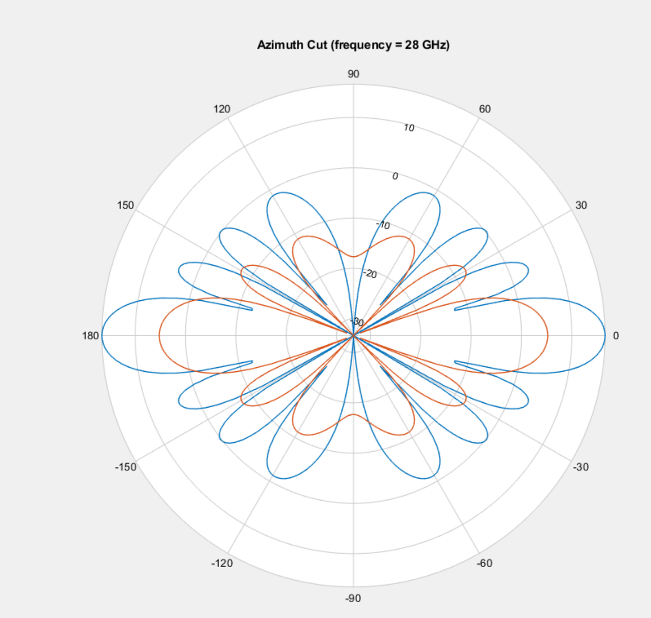
Plotting the Patterns

- MATLAB has excellent routines for 3D patterns
- Note that this plots directivity not array factor

```
sv = phased.SteeringVector('SensorArray',arr);
ang0 = [0; 0];
sv.release();
u0 = sv(fc, ang0);
u0 = u0 / norm(u0);
```




```
% We can plot the directivity in a 3D plot
arr.pattern(fc,'Weights', u0);
```




```
elPlot = [0 45];
arr.patternAzimuth(fc, elPlot, 'Weights', u0);
```

Element Gain

- Above analysis assumes each element is omni-directional
- Each antenna element may also have gain.
- Assume all elements of an array are identical and have same orientation
- Pattern multiplication theorem: The frequency response of a single path channel is:

$$\boxed{\boldsymbol{h}(\omega) = g(\omega) A_E(\Omega) \boldsymbol{u}(\Omega)}$$

Freq response @reference

Element gain

Spatial signature

- Resulting array factor (in linear scale): $AF(\Omega, \Omega_0) = AF_{\text{iso}}(\Omega, \Omega_0) A_E(\Omega)$
  - $AF_{\text{iso}}(\Omega, \Omega_0) = \frac{1}{\sqrt{M}} \boldsymbol{u}^*(\Omega_0) \boldsymbol{u}(\Omega)$ = array factor with isotropic elements

Example: URA with Patch Elements

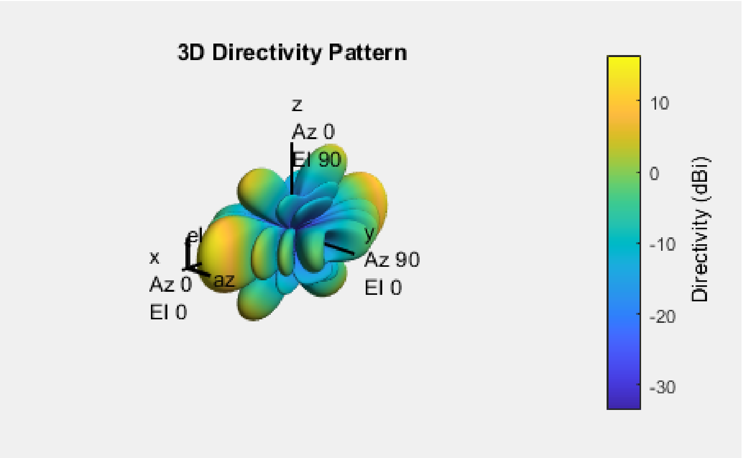
- ❑ Example 4x8 URA
- ❑ Add patch element
  - Element normal in +x direction
  - Peak element gain ≈ 8 dBi
  - Adds to the total array gain

Isotropic elements

4 x 8 URA

Peak directivity ≈ 15 dB

Gain in both positive and negative x direction

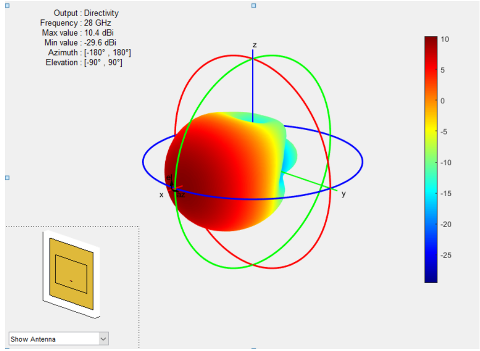


Patch elements

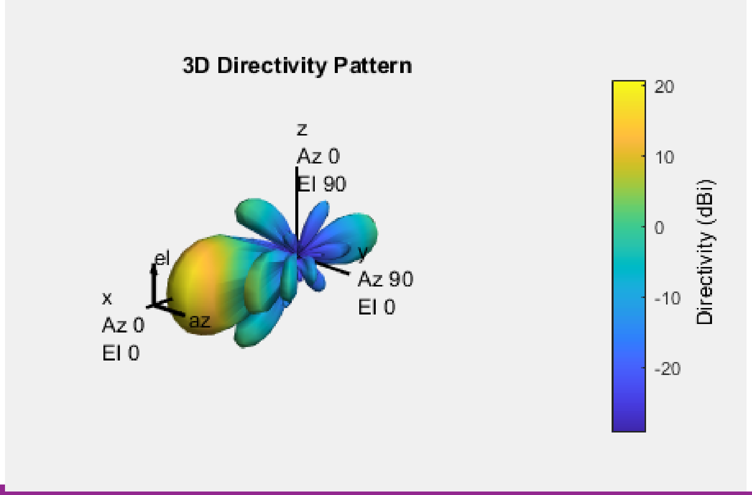
4 x 8 URA

Peak directivity ≈ 21 dB

Gain low in negative x direction

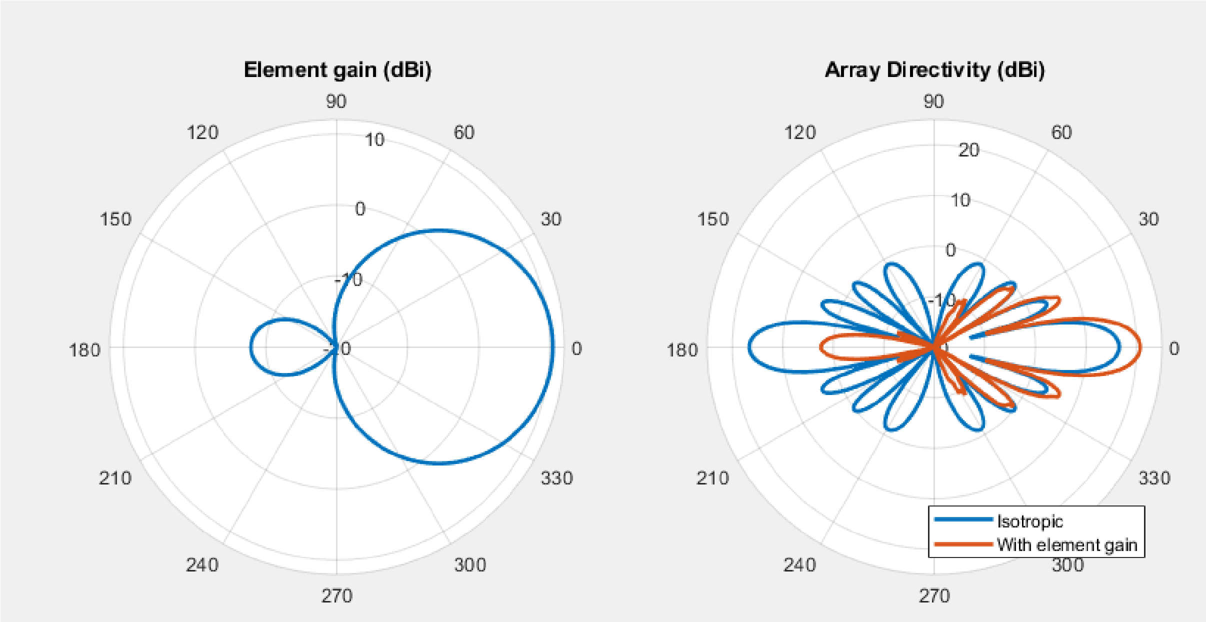


Element pattern



Example: URA with Patch Elements in 2D

- Pattern multiplication in 2D
- Element gain increases directivity
- Note: MATLAB plots directivity
  - Does not plot array gain
  - Directivity is array gain normalized to one



In-Class Problem: Simulating BF Mismatch

- Continue simulation but with BF mismatch

Outline

- ❑Antenna arrays and the Spatial Signature
- ❑Receive Beamforming and SNR Gain
- ❑Array Factor
- ❑Multiple paths and Diversity
- ❑Transmit Beamforming

Multiple Paths

- Easy to extend channel response to multiple paths
- Each path adds a term with a spatial signature
- Time-domain model

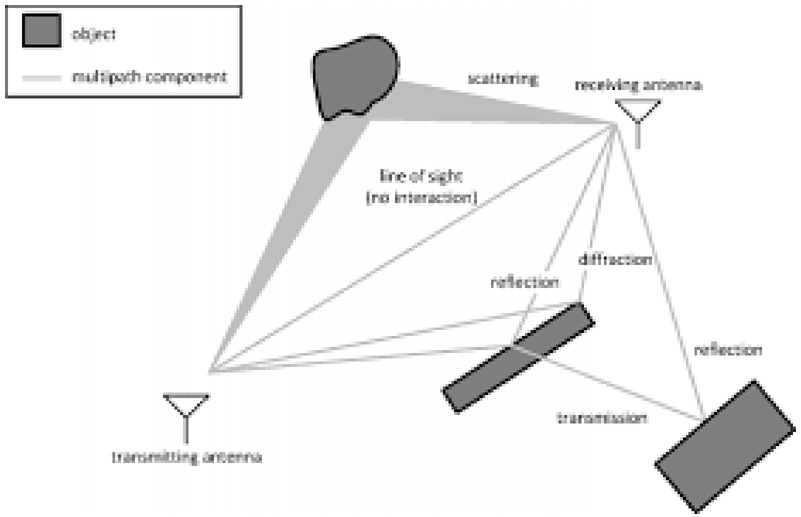
$$\boldsymbol{r}(t) = \sum_{\ell=1}^{L} g_\ell e^{j\omega_\ell t}\, \boldsymbol{u}(\Omega_\ell) x(t-\tau_\ell) + \boldsymbol{n}(t)$$

Doppler shift

Delay

AoA

Complex gain



Time-Varying Frequency Response

- Apply input $x(t) = e^{j\omega t}$
- RX vector is $\boldsymbol{r}(t) = \boldsymbol{h}(t,\omega)x(t)$
- Time-varying frequency response
- $\boldsymbol{h}(t,\omega) = \sum_{\ell=1}^{L} g_\ell e^{j\omega_\ell t - j\omega\tau_\ell}\,\boldsymbol{u}(\Omega_\ell)$
- Vector channel response

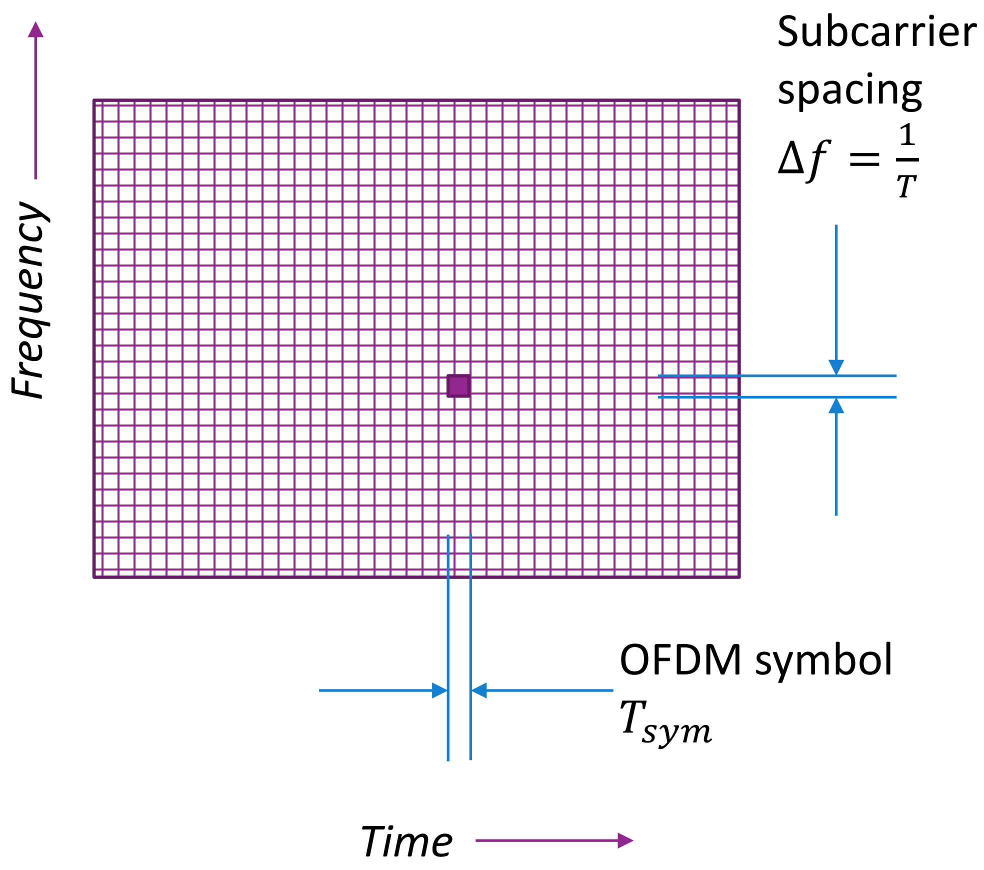
Time-Varying Frequency Response

- Multipath channel: $\boldsymbol{r}(t) = \sum_{\ell=1}^{L} g_\ell e^{j\omega_\ell t} \boldsymbol{u}(\Omega_\ell)\, x(t - \tau_\ell)$
- Consider exponential scalar input: $x(t) = e^{j\omega t}$
- Vector output is: $\boldsymbol{r}(t) = \boldsymbol{h}(t,\omega)x(t)$
- Time-varying frequency response

$$\boldsymbol{h}(t,\omega) = \sum_{\ell=1}^{L} g_\ell e^{j(\omega_\ell t - \omega \tau_\ell)} \boldsymbol{u}(\Omega_\ell)$$

- May also write: $\boldsymbol{h}(t,f) = \boldsymbol{h}(t, 2\pi f)$

OFDM Time-Frequency Grid



- Recall OFDM from earlier lecture
- Divide channel into sub-carriers and OFDM symbols
  - Resource element: One time-frequency point
- Data is transmitted is an array: $X[n,k]$
  - k = OFDM symbol index, n = subcarrier index
  - One complex value per RE.
- Receive a vector:

$$\boldsymbol{Y}[n,k] = \left[Y_1[n,k], \dots, Y_M[n,k]\right]^T$$

  - One complex symbol per antenna per RE

OFDM Channel with Multiple RX Antennas

- OFDM channel acts as multiplication:
  Under normal operation (delay spread is contained in CP):

$$\boldsymbol{Y}[k,n] = \boldsymbol{H}[k,n] \;\; X[k,n]$$

RX symbol vectors — Vector channel — TX symbols

- OFDM channel gains can be computed from the multi-path components

$$\boxed{\boldsymbol{H}[k,n] = \sum_{\ell=1}^{L} \sqrt{E_\ell} e^{-2\pi j \, (Tkf_\ell + Sn\tau_\ell + \phi_\ell)} \, \boldsymbol{u}(\Omega_\ell)}$$

  - T = OFDM symbol time, S = sub-carrier spacing
  - For each path: f_ℓ =Doppler shift, τ_ℓ =Delay, ϕ_ℓ = phase of path, E_ℓ = path received energy

Time Scales

- Consider vector channel response

$$\boldsymbol{h}(t,\omega) = \sum_{\ell=1}^{L} g_\ell e^{j\omega_\ell t - j\omega\tau_\ell}\, \boldsymbol{u}(\Omega_\ell)$$

- Large scale parameters: Change slowly
  - Gain g_ℓ and angles Ω_ℓ
  - Depend on geometry and large obstacles.
- Small scale parameters: Change rapidly
  - $\omega\tau_\ell$: Changes over frequency on order of inverse delay spread
  - $\omega_\ell t$: Changes over time on order of Doppler spread

RX Correlation

- How correlated are two different antennas?
  - Related to diversity gain
- Covariance matrix

$$\boldsymbol{Q} = cov[\boldsymbol{h}(t,\omega)] = E(\boldsymbol{h}(t,\omega) - \boldsymbol{\mu})(\boldsymbol{h}(t,\omega) - \boldsymbol{\mu})^*$$

- Typically fix AoA and path gains, average over ω and t
- Averaging over time and frequency: $E\boldsymbol{h}(t,\omega) = 0$ and

$$\boxed{\boldsymbol{Q} = \sum_{\ell=1}^{L} |g_\ell|^2 \boldsymbol{u}(\Omega_\ell)\boldsymbol{u}(\Omega_\ell)^*}$$

  - Proof on board

Correlation with Random AoAs

- Suppose:
  - ULA with M elements
  - L large. Total power gain G
  - AoAs spread θ had pdf $p(\theta)$
- Then:

$$Q_{km} = G \int_0^{2\pi} p(\theta) e^{ikd(k-m)\cos\theta} d\theta$$

Correlation with Uniform AoAs

- If θ uniform $[0,2\pi]$
- Then:

$$Q_{jm} = \frac{G}{2\pi}\int_0^{2\pi} e^{ikd(j-m)\cos\theta}\, d\theta = J_0\left(\frac{2\pi d_{jm}}{\lambda}\right)$$

  - $d_{jm} = d(j-m)$ distance between antennas
  - $J_0(x) =$ Bessel function
- Become uncorrelated when $d_{jm} \gg \lambda$
- Need more spacing for smaller range of angles

Diversity Gain

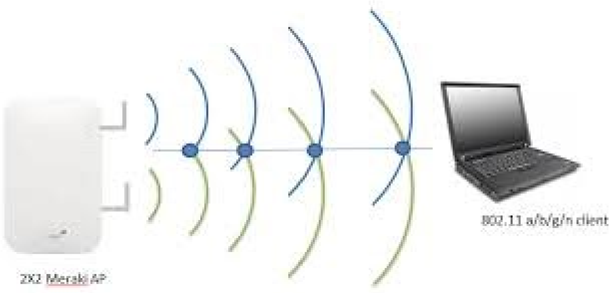
- Peak gain does not depend on antenna size
- High diversity gain requires wide separation
- Example:
  - $f_c = 3$ GHz
  - $\lambda = 10$ cm
  - Antenna separation $10\lambda = 1$ m
  - Possible in a cellular tower.
  - Not possible in a handset

Outline

- ❑ Antenna arrays and the Spatial Signature
- ❑ Receive Beamforming and SNR Gain
- ❑ Array Factor
- ❑ Multiple paths and Diversity
- ❑ Transmit Beamforming

Multiple TX antennas

- MISO channel
  - Multiple input single output
  - M TX antennas, 1 RX antennas
  - Transmit vector: $\boldsymbol{x}(t) = \left(x_1(t), \dots, x_M(t)\right)^T$
  - Scalar RX: $r(t)$



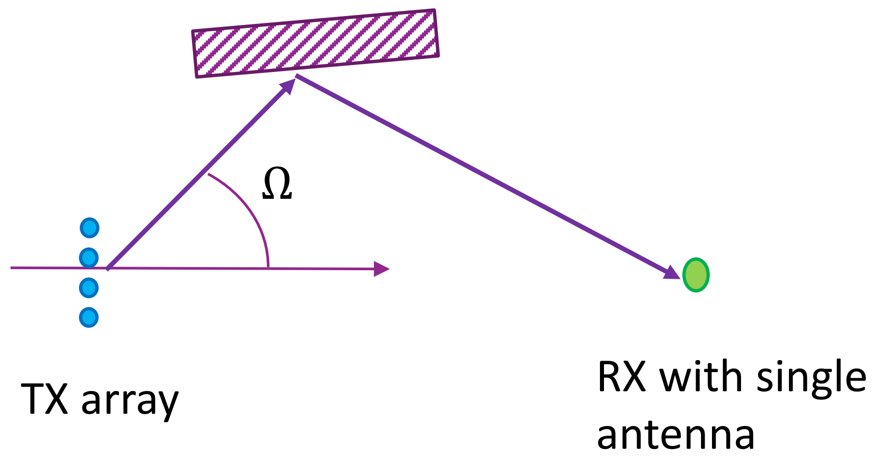
- Most of the theory is identical to the SIMO channel

Single Path Channel

- First consider single path channel
- Similar to SIMO case, RX signal is:

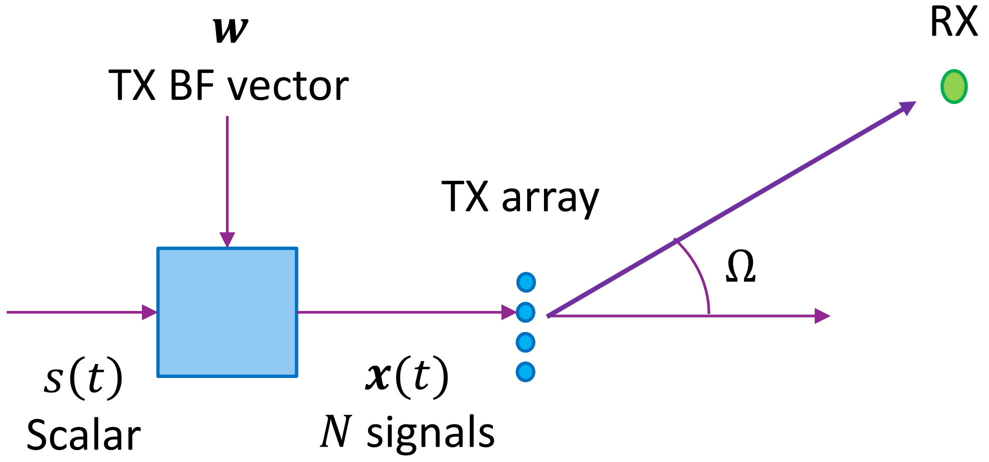
$$r(t) = g_0 \boldsymbol{u}^*(\Omega)\boldsymbol{x}(t-\tau)$$

  - g_0 path gain
  - Ω = angle of departure
  - τ = path delay
  - $\boldsymbol{u}^*(\Omega)$ spatial signature
- TX and RX spatial signatures are identical
  - Except you apply the conjugate transpose



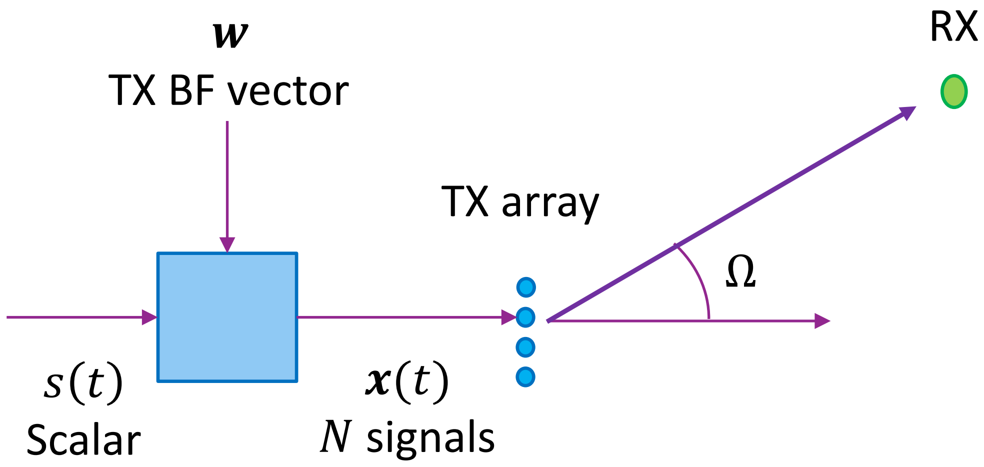
TX Beamforming

- RX signal is: $r(t) = g_0 \boldsymbol{u}^*(\Omega)\boldsymbol{x}(t-\tau) + n(t)$
- TX beamforming
  - Input scalar information signal $s(t)$
  - Create vector signal to antennas: $\boldsymbol{x}(t) = \boldsymbol{w}\, s(t)$
  - $\boldsymbol{w}$ is called the TX beamforming vector
- Also called pre-coding



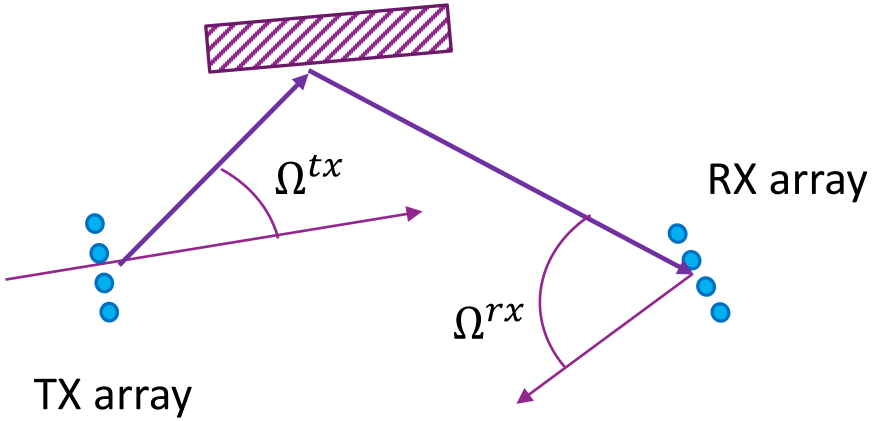
MRC TX Beamforming

- RX signal is: $r(t) = g_0 \boldsymbol{u}^*(\Omega)\boldsymbol{x}(t-\tau) + n(t)$
- Analysis is identical to SIMO case
- MRC TX BF vector: $\widehat{\boldsymbol{w}} = \frac{1}{\sqrt{N}}\boldsymbol{u}(\Omega)$
  - Align with AoD
- SNR gain $= N$
- Define and compute Array Factor similarly
- Also define multi-path channel



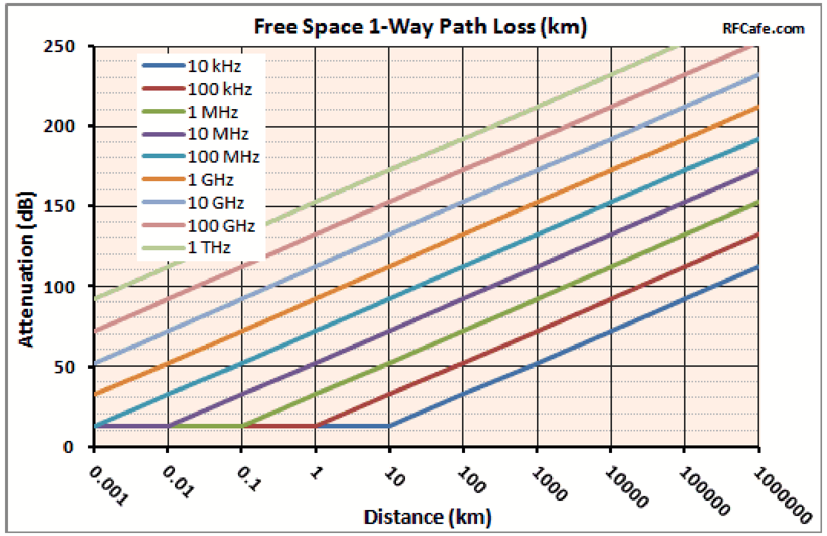
Beamforming and Channel Estimation

- Key issue for beamforming: Channel estimation
- TX and RX beamforming require that channel is known
- We will discuss many of these concepts later
  - Reference signals
  - Channel feedback
  - Channel tracking
  - Beam management
  - Spatial equalization



Friis' Law and MmWave

- Recall Friis' Law: $\frac{P_r}{P_t} = D_1 D_2 \left(\frac{\lambda}{4\pi R}\right)^2$
- Isotropic path loss decreases with λ^2
- Millimeter Wave systems: Increases f_c^2
  - Decreases λ^2 $\Rightarrow$ Increase path loss
  - Compensate isotropic path loss with directivity, D_i
- Fix aperture A_1 on TX side, A_2 on receiver side
  - Can fit $N_i = \frac{cA_i}{\lambda^2}$ antennas on each side
  - Leads to directivity: $\mathrm{D_i} \propto N_i \propto \frac{A_i}{\lambda^2}$
- Can compensate isotropic path loss with directivity



Friis' Law and MmWave

Condition	Directivity scaling	Path loss scaling
No beamforming	D_i constant	$PL \propto f_c^2$
Beamforming on one side (TX or RX)	$D_1 \propto f_c^2,\ D_2$ constant	PL constant
Beamforming on both sides (TX and RX)	$D_1, D_2 \propto f_c^2$	$PL \propto f_c^{-2}$

- Friis' Law: $\frac{P_r}{P_t} = D_1 D_2 \left(\frac{\lambda}{4\pi R}\right)^2$
- Conclusions: With a fixed aperture and beamforming
  - Isotropic path loss can be overcome
- But systems need very directive beams
  - Raises many other issues. E.g. Channel tracking, processing, …