Problems: Coding and Capacity on Fading Channels
ECE-GY 6023. Wireless Communications

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1. *Slow vs. fast fading:*. For each scenario below state whether the variations would likely be slow or fast fading relative to the coding block. Use reasonable assumptions and explain your reasoning. There is no single correct answer.

   (a) A 5G NR base stations transmits over a channel with a 100 ns delay spread, to a UE moving at $v = 30\,\mathrm{m/s}$ with a 180° angular spread. The carrier frequency is $f_c = 28\,\mathrm{GHz}$. The transmission is over a 100 MHz bandwidth in 125 µs slots.

   (b) A UAV is connected to a ground base station via point-to-point link with a line-of-sight. So, there is no multipath fading. But the UAV rotates 360° about once a second. The beamwidth of the UAV antenna element is 60° and packets are transmitted once every 1 ms.

2. *Error rate on uncoded modulation*:

   (a) Use any reference to find the symbol error rate (SER) of 16-QAM as a function of the SNR $\gamma_s = E_s/N_0$. Your expression will have a Q-function.

   (b) Find the SNR γ_s requred for a SER of $(10)^{-3}$ assuming a constant channel. You can use MATLAB to invert the Q-function.

   (c) Suppose that the channel is Rayleigh fading, so γ_s is exponentially distributed. Find the average SNR, $\mathbb{E}(\gamma_s)$ so that the average SER is $(10)^{-3}$.

3. *Slow fading and outage probability:* An access point is installed in an office area with four rooms. The path loss from the access point to each room and the percentage of users in each room are as follows:

Room	Path loss [dB]	Fraction users
1	60	0.6
2	80	0.3
3	90	0.06
4	100	0.04

   The AP has a transmit power of 15 dBm and bandwidth of 18 MHz. The thermal noise at the receivers, including noise figure is $-165\,\mathrm{dBm/Hz}$.

   (a) If there is no fading, what SNR can be guaranteed to at least 95% of the users?

(b) Now suppose that, at each location, there is Rayleigh fading that can be modeled as flat over the transmissions. Write an expression for the CDF of the SNR including variation in both location and fading.

(c) What is the SNR that can be guaranteed to at least 95% of the users if we need to account for slow fading? You can use MATLAB to invert the expression in part (b).

4. *Ergodic capacity:* A channel has two paths. One path would be received at power, P_1 and delay τ_1, and the second path would be received at power P_2 and delay τ_2 where $\tau_2 > \tau_1$. Suppose you signal over a bandwidth $W \gg 1/(\tau_2 - \tau_1)$ and noise power spectral density is N_0.

   (a) What is the average SNR over the band?

   (b) What is the ergodic capacity over the band?

   (c) Evaluate the expressions in (a) and (b) with $P_1/(WN_0) = 8\,\text{dB}$ and $P_2/(WN_0) = 5\,\text{dB}$.

5. *LLRs:* For each of the following channels, find the log likelihood ratio (LLR):

$$L(r) = \log \frac{p(r|c=1)}{p(r|c=0)}$$

   for the following channels:

   (a) Real-valued binary channel with fading:

$$r = Ax + w, \quad w \sim \mathcal{N}(0, N_0/2), \quad x = \begin{cases} \sqrt{E_x/2} & \text{if } c = 1, \\ -\sqrt{E_x/2} & \text{if } c = 0. \end{cases}$$

   The LLR L should depend on A and N_0.

   (b) Binary symmetric channel:

$$r = c + w \pmod 2, \quad w = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$$

   Thus, $r \in \{0, 1\}$ where there is a bit error with probability p.

   (c) Non-coherent channel:

$$r = \begin{cases} h + n & \text{when } c = 1 \\ n & \text{when } c = 0, \end{cases} \quad h \sim \mathcal{CN}(0, E_s), \; n \sim \mathcal{CN}(0, N_0).$$

6. *Bitwise likelihood*: Suppose that two bits (c_0, c_1) are modulated to a 4-PAM constellation (the real or imaginary component of a 16-QAM constellation):

$$r = x + n, \quad n \sim \mathcal{N}(0, N_0/2),$$

   where the transmitted symbol

$$x = \begin{cases} -3A & \text{if } (c_0, c_1) = (00) \\ -A & \text{if } (c_0, c_1) = (01) \\ A & \text{if } (c_0, c_1) = (11) \\ 3A & \text{if } (c_0, c_1) = (10) \end{cases}$$

   Assume all the transmitted bits are equally likely.

(a) Given a symbol energy, E_s, find A such that $\mathbb{E}|x|^2 = E_s/2$.

(b) Find the bitwise LLR for c_0:

$$L_0(r) = \log \frac{p(r|c_0 = 1)}{p(r|c_0 = 1)}.$$

Use total probability

$$p(r|c_0) = \frac{1}{2}\left[p(r|c_0, c_1 = 1) + p(r|c_0, c_1 = 0)\right].$$

Find the bitwise LLR for c_1 as well.

7. *Row-column interleavers:* One simple way of doing interleaving is as follows. The input is a sequence of bits of length MN for some parameters M and N. We read the bits into an $M \times N$ array, one row at a time. Then, we read out the bits one column at a time. If two bits are adjacent on the input what is the minimum separation on the output?