Propagation and Link-Budget Calculations. In-Class Exercises

These exercises accompany the lectures for this unit and should be done at the end of each section.

Problem 1: Adding Noise and Measuring SNR

In this problem, we will simulate a simple cascade of two receiver elements:

- An LNA with gain `GLna` and noise figure `NFLna`
- A mixer with unity gain and noise figure `NFmix`

The received signal has power `Prx` and bandwidth `B`.

```
Prx = -80;      % RX power in dBm
NFLna = 6;      % Noise figure in the LNA in dB
GLna = 15;      % LNA gain in dB
NFmix = 13;     % mixer noise figure
B = 18e6;       % Bandwidth in Hz
```

Compute and print the effective noise figure.

```
% TODO
%    NFeff = ...

% Effective noise figure
NFeff = db2pow(NFLna) + (db2pow(NFmix)-1)/db2pow(GLna);
NFeff = pow2db(NFeff);

fprintf(1, 'Effective NF (dB) = %f\n', NFeff)
```

```
Effective NF (dB) = 6.609040
```

Assume the energy per sample in linear scale is `Es = Prx/B`. Compute the SNR per sample, `EsN0`, including the effective noise figure.

```
% TODO
%    Es = ...
%    EsN0 = ...

% Noise energy per sample
T = 290;
k=physconst('Boltzmann');
EkT = 10*log10(k*T);
Enoise = EkT + NF;

% Energy per symbol
Es = Prx-30-pow2db(B);

% SNR
EsN0 = Es - EkT - NFeff;
```

```
fprintf(1, 'EsN0 (dB) = %7.2f\n', EsN0);
```

```
EsN0 (dB) =   14.81
```

We will now simulate this system. Generate `nsym` QPSK modulation symbols and scale them to the correct energy per symbol. Place the results in a vector `s`.

```
% Parameters
modRate = 4;    % num bits per symbol (4=16 QAM)
M = 2^modRate;  % QAM order
nsym = 10000;    % num symbols

% TODO
%    sym = ...

% Generate data
nbits = nsym*modRate;
bits = randi([0,1],nbits,1);
sym = qammod(bits,M,'InputType','bit','UnitAveragePower',true);

% Scale the received symbols
Es = db2pow(Prx - 30)/B;
s = sqrt(Es)*sym;
```

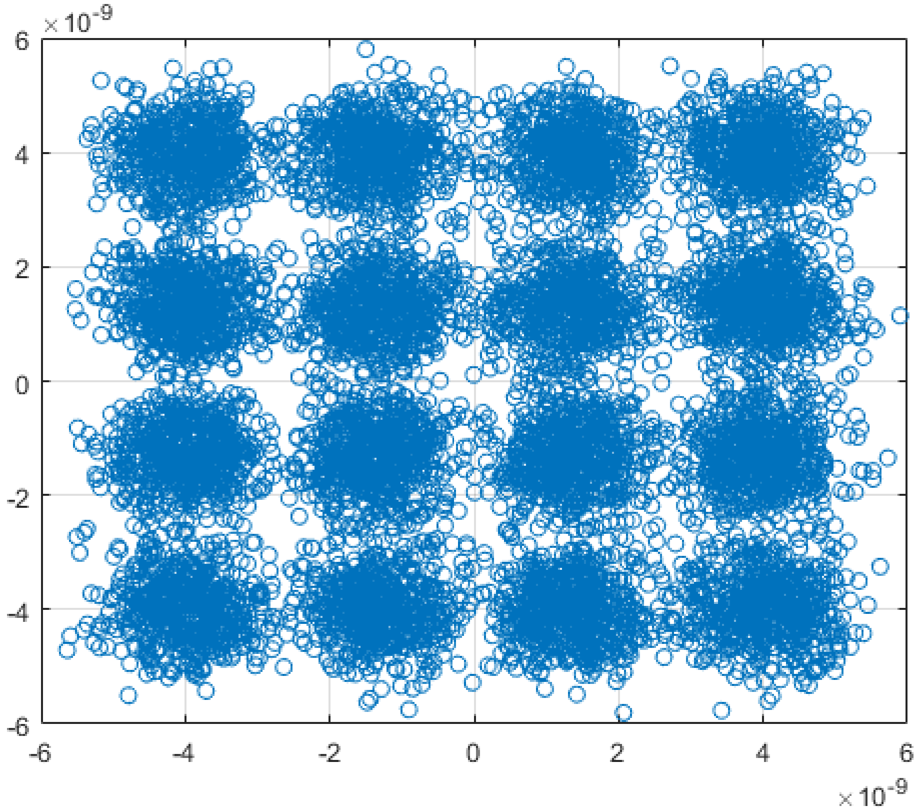
Now simulate the receiver:

- Add the thermal noise of the LNA
- Scale the signal by the LNA gain
- Add the thermal noise of the mixer

For the thermal noise in the LNA and mixer use the `comm.ThermalNoise` class. Store the results in a vector `r`. Plot the received constellation.

```
% TODO:
%    r = ...

% Add thermal noise
tnLna = comm.ThermalNoise('NoiseMethod', 'Noise figure', ...
    'NoiseFigure', NFLna, 'Add290KAntennaNoise', true);

% Add the noise
r1 = tnLna.step(s);

% Model the LNA gain
G = db2mag(Glna);
r2 = G*r1;

% Add the mixer noise
tnMix = comm.ThermalNoise('NoiseMethod', 'Noise figure', ...
    'NoiseFigure', NFmix);
r = tnMix.step(r2);

plot(real(r), imag(r), 'o');
```

```
grid on;
```



Compute the SNR

```
EsNoMeas = 10*log10( E|G*s|^2/ E|r - G*s|^2 )
```

where `G` is the receiver gain in linear scale and `E(.) denotes the average value.  Measure and print EsN0Meas and compare with the expected value.`

```
% TODO
%   EsN0Meas = ...
rerr = r - G*s;
EsN0Meas = abs(G)^2*mean(abs(s).^2)/mean(abs(rerr).^2);
EsN0Meas = 10*log10(EsN0Meas);
fprintf(1, 'SNR measured = %7.2f\n', EsN0Meas);
```

```
SNR measured =   14.76
```

```
fprintf(1, 'SNR expected = %7.2f\n', EsN0);
```

```
SNR expected =   14.81
```

In many cases, we do not know the gain `G, so it has to be estimated to compute the SNR.`
Suppose that we have a model:

```
r = G*s + d,   d = CN(0, N0)
```

where `G` is the unknown gain, and `N0` is the unknown noise variance. We can estimate the gain and noise variance via linear regression:

```
Ghat = (r'*s)/(s'*s);
N0hat = (1/n)*||r - Ghat*s||^2 = (1/n)*(r'*r - (r'*s)^2/(s'*s))
Eshat = (1/n)*|Ghat|^2*||s||^2
```

This gives and SNR estimate:

```
EsN0hat = Eshat / N0hat
        = |Ghat|^2*(s'*s) / (r'*r - |r'*s|^2/(s'*s))
        = |r'*s|^2 / (s'*s) / (r'*r - |r'*s|^2/(s'*s))
        = |r'*s|^2 / ( (r'*r)*(s'*s) - |r'*s|^2 )
        = rho / (1 - rho)
```

where `rho` is the correlation coefficient:

```
rho = |r'*s|^2/(r'*r)*(s'*s)
```

Use this estimate to estimate the SNR.

```
% TODO
%  EsN0hat = ...
rho = real( abs(r'*s)^2/ (r'*r) / (s'*s) );
EsN0hat = 10*log10(rho/(1-rho));
fprintf(1, 'SNR estimated = %7.2f\n', EsN0hat);
```

```
SNR estimated =   14.76
```

Problem 2. Estimating the SNR Requirements for LTE

In this problem, we will see how the performance of a commercial LTE system comparse to the Shannon theory. The LTE standard 36.942 provides a number of details of how to evaluate the performance of the system in various scenarios. You can find any of the 3GPP standards by googling, e.g. "3GPP 36.942". Below is a table from that document that shows the code performance as a function of the SNR for the uplink and downlink.

Table A.2 Look-Up-Table of UL and DL Throughput vs. SNIR for Baseline E-UTRA Coexistence Studies

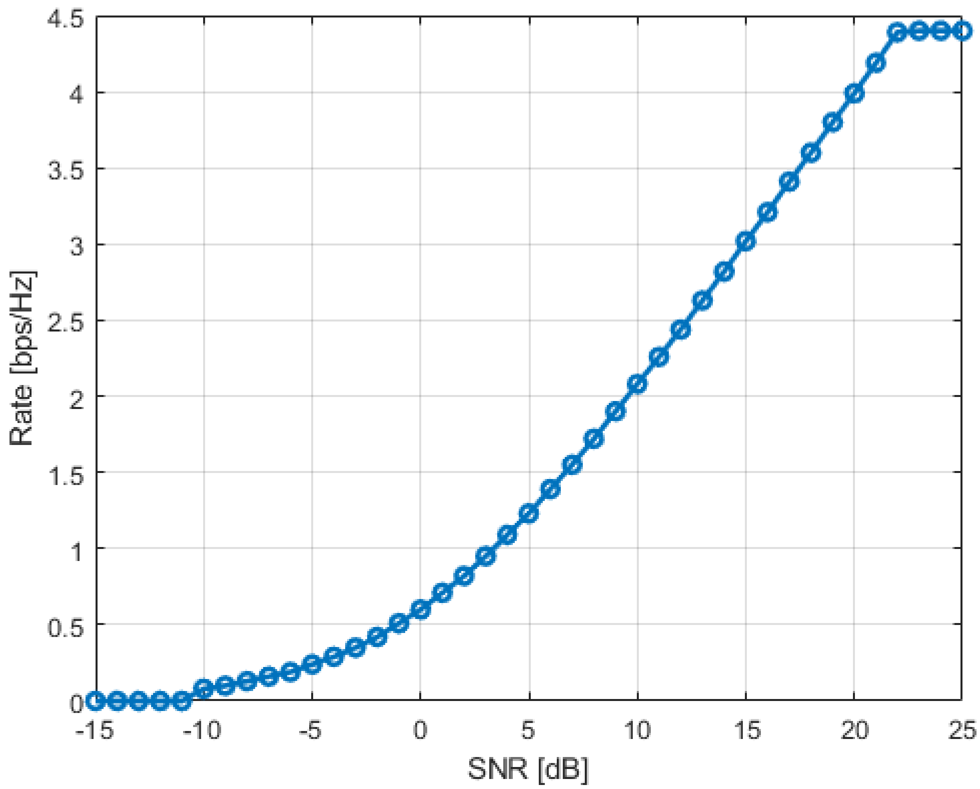
SNIR	Throughput					SNIR	Throughput			
	bps/Hz		kbps per 375kHz RB				bps/Hz		kbps per 375kHz RB	
dB	DL	UL	DL	UL		dB	DL	UL	DL	UL
-15	0	0	0	0		6	1.39	0.93	521	347
-14	0	0	0	0		7	1.55	1.04	582	388
-13	0	0	0	0		8	1.72	1.15	646	430
-12	0	0	0	0		9	1.90	1.26	711	474
-11	0	0	0	0		10	2.08	1.38	778	519
-10	0.08	0.06	31	21		11	2.26	1.51	847	565
-9	0.10	0.07	38	26		12	2.44	1.63	917	611
-8	0.13	0.08	48	32		13	2.63	1.76	988	658
-7	0.16	0.10	59	39		14	2.82	1.88	1059	706
-6	0.19	0.13	73	48		15	3.02	2.00	1131	750
-5	0.24	0.16	89	59		16	3.21	2.00	1204	750
-4	0.29	0.19	109	73		17	3.41	2.00	1277	750
-3	0.35	0.23	132	88		18	3.60	2.00	1350	750
-2	0.42	0.28	159	106		19	3.80	2.00	1424	750
-1	0.51	0.34	190	127		20	3.99	2.00	1498	750
0	0.60	0.40	225	150		21	4.19	2.00	1572	750
1	0.71	0.47	265	176		22	4.39	2.00	1646	750
2	0.82	0.55	308	206		23	4.40	2.00	1650	750
3	0.95	0.63	356	237		24	4.40	2.00	1650	750
4	1.09	0.72	408	272		25	4.40	2.00	1650	750
5	1.23	0.82	463	309						

In the github folder, you will find an Excel file with this data. You can read it via the command which will load the data into a table `T`.

```
T = readtable('lte_mcs.xlsx');
```

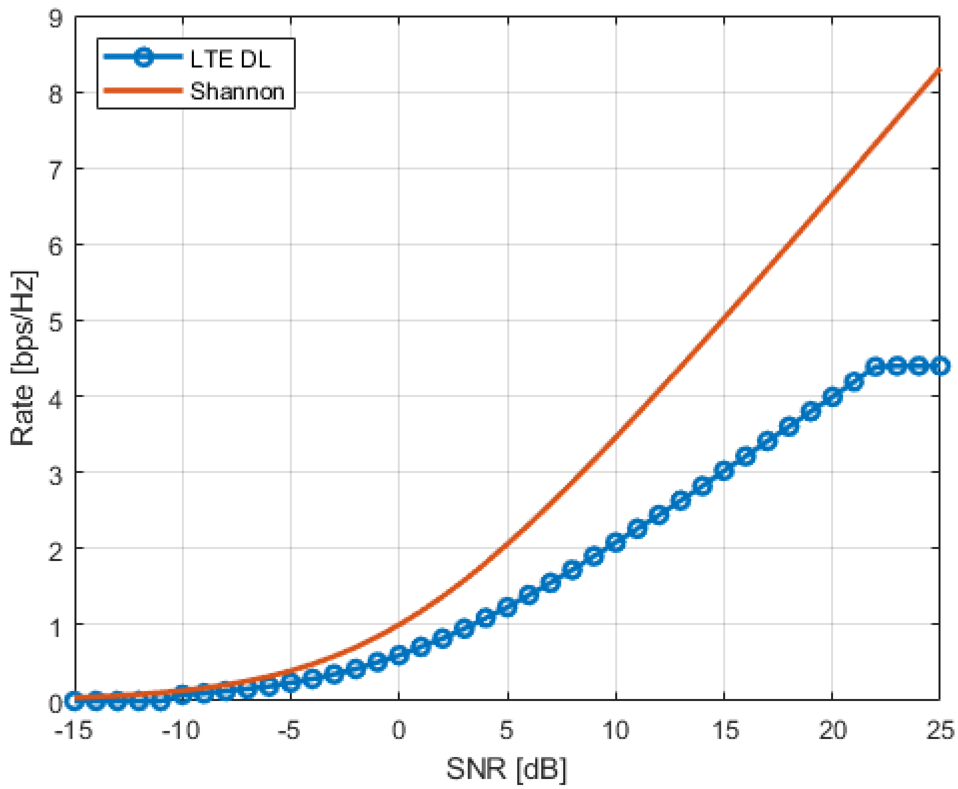
Now, read the SNR and downlink spectral efficiency from the table into vectors `snr` and `se_dl`. You can use the `table2array` function. Plot the `se_dl` vs. `snr`.

```
% TODO
%  snr = ...
%  se_dl = ...
snr = table2array( T(:,"SNIR") );
se_dl = table2array( T(:,"SE_DL") );

plot(snr, se_dl, 'o-', 'linewidth', 2);
xlabel('SNR [dB]');
ylabel('Rate [bps/Hz]');
grid on;
```



On the same plot as above, plot the Shannon rate vs. SNR. You should see that the LTE value is lower.

```
% TODO
%    se_shannon = ...
se_shannon = log2(1 + 10.^(0.1*snr));

plot(snr, se_dl, 'o-', 'linewidth', 2);
hold on;
plot(snr, se_shannon, '-', 'linewidth', 2);
xlabel('SNR [dB]');
ylabel('Rate [bps/Hz]');
grid on;
hold off;
legend('LTE DL', 'Shannon', 'Location', 'northwest');
```



Fit a model for the LTE DL rate:

```
se_dl_hat = alpha*log2(1 + 10.^(0.1*snr) )
```

where `alpha` is a parameter that we will fit. Fit this model using the data for SNR from -10 to 21 dB. To fit a model of the form `y_i = alpha*x_i`, you can use the regression formula:

```
alpha = \sum_i (x_i*y_i) / \sum_i x_i^2
```

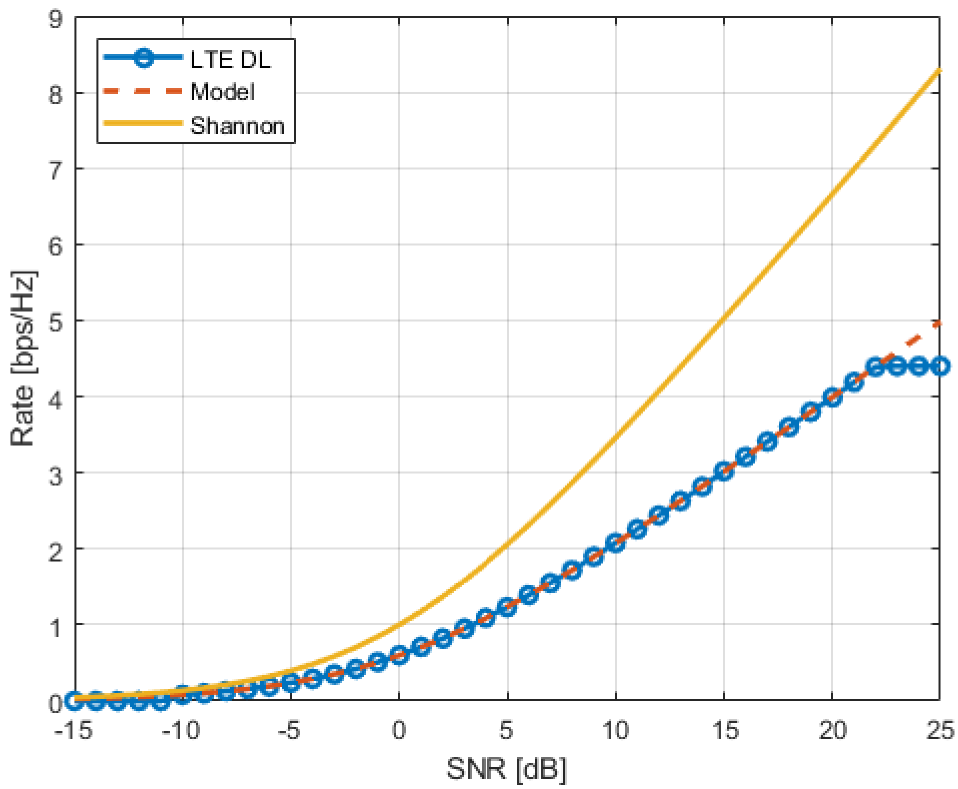
Print alpha. Also add the line `se_dl_hat` vs. `snr` to the previous plot.

```
% TODO:
%   alpha = ...
%   se_dl_hat = ...
I = find((snr >= -10) & (snr <= 20));
x = log2(1 + 10.^(0.1*snr(I)));
y = se_dl(I);
alpha = (x'*y)/(x'*x);
se_dl_hat = a*log2(1 + 10.^(0.1*snr));

fprintf(1, 'alpha = %f', alpha);
```

```
alpha = 0.599930
```

```
plot(snr, se_dl, 'o-', 'linewidth', 2);
hold on;
plot(snr, se_dl_hat, '--', 'linewidth', 2);
plot(snr, se_shannon, '-', 'linewidth', 2);
xlabel('SNR [dB]');
```

```
ylabel('Rate [bps/Hz]');
grid on;
hold off;
legend('LTE DL', 'Model', 'Shannon', 'Location', 'northwest');
```



Problem 3: Simulating Path Loss Variations

Suppose the TX power is `Ptx` and the path loss can be modeled as a random variable as follows:

- The channel is either LOS or NLOS with probability `ProbLos` and `1-ProbLos, respectively.`
- If the channel is LOS, the path loss is lognormally distributed with mean `PLLos` and standard deviation `stdLos`.
- If the channel is NLOS, the path loss is lognormally distributed with mean `PLNlos` and standard deviation `stdNlos`.

Generate `n=10000` channel instances and plot the histogram of the received power. You may use the MATLAB `histogram` function.

```
Ptx = 15;        % Tx power in dBm
ProbLos = 0.4;   % LOS probability
PLNlos = 80;     % Path loss for LOS (dB)
PLLos = 100;     % Path loss for NLOS (dB)
stdLos = 8;      % Path loss std dev for LOS (dB)
stdNlos = 4;     % Path loss std dev for LOS (dB)
n = 10000;       % number of samples

% TODO
%   Prx = ...
%   histogram(Prx, ...);
ulos = (rand(n,1) < ProbLos);
```

```
PLnom = PLLos*ulos + PLNlos*(1-ulos);
std = stdLos*ulos + stdNlos*(1-ulos);
Prx = Ptx - PLnom + std.*randn(n,1);

histogram(Prx,'BinWidth',2,'Normalization','pdf');
grid on;
xlabel('RX power (dBm)');
ylabel('Probability');
```