Problems: Adaptive Modulation and Coding
ECE-GY 6023. Wireless Communications

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1. *MCS selection:* A system has four MCS selections with minimum required SNRs as shown:

MCS	1	2	3	4
Min SNR [dB]	0	4	8	12

   (a) Suppose that the SNR in dB, γ, is unknown and can be modeled as Gaussian variable with mean 6 dB and standard deviation of 2 dB. What is the probability that $\gamma > 8$ dB, the required SNR for MCS 3.

   (b) The TX attempts MCS 3 and it fails, meaning $\gamma \leq 8$ dB. If it now attempts MCS 2, what is the probability that it will pass assuming the channel has not changed.

2. *Markovian errors:* In this problem, we will show how to use Markov processes to model error probabilities on correlated channels. As a simple example, suppose that a channel in time slot k can be modeled as being in one of two states: a good state ($X_k = 1$), and a bad state ($X_k = 0$). Assume X_k is Markov with transition probability matrix

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}.$$

   A transmitter sends packets in each time slot. Let $Y_k = 0$ or 1 be the indicator that a packet fails or passes in time slot k. Assume,

$$P(Y_k = 1 | X_k = 1) = 0.8, \quad P(Y_k = 1 | X_k = 0) = 0.4.$$

   Assume that, given the X_k's, the Y_k's are independent.

   (a) Let $\alpha_k(i) = P(X_k = i)$. Find the recursion for the values $\alpha_{k+1}(i)$ in terms of the values $\alpha_k(j)$.

   (b) Let

$$\alpha_k^0(i) = P(X_k = i | Y_0 = 0, \ldots, Y_{k-1} = 0).$$

   That is, $\alpha_k^0(i)$ is the probability that $X_k = i$ given that the previous $k-1$ transmissions have failed. Find the recursion for $\alpha_{k+1}^0(i)$ in terms of the values $\alpha_k^0(i)$.

   (c) Let T be the time,

$$T = \min\{\ k\ | Y_k = 1\ \},$$

   which is the index of the first slot that the packet passes. Suppose that $X_0 = 0$. Write a simple MATLAB program to compute $P(T = k)$ for $k = 0, 1, \ldots, 9$ using the above recursions.

3. *Multi-Process ARQ timeline:* Suppose that a gNB wants to send $N = 10$ packet data units (PDUs). The PDUs are indexed $n = 0, 1, \ldots, N-1$. In each slot, it attempts to send one PDU beginning in slot $k = 0$ starting with PDU 0. There are $K = 4$ parallel HARQ processes. Suppose that the transmissions fails in slots $k = 5, 6$ and 7 and passes in all other slots.

   (a) For each PDU, indicate the fist slot it is correctly decoded at the receiver.

   (b) Suppose the receiver only releases decoded the PDUs in order to the higher layer. So, for example, it holds PDU 3 back until it receives PDU 2. Also, there is a fixed delay of 3 slots from the time of transmission to the PDU being available at the receiver for higher layers. When do the PDUs arrive at the higher layer?

4. *TB size:* Suppose that a 64 kbps voice over IP (VoIP) system transmits frames once every 20 ms. Each voice frame also requires a 20 B IP header, 20 B UDP header, and 24 bits CRC.

   (a) How many bits are in each voice frame?

   (b) Suppose the data is transmitted in an NR-like system with 14 OFDM symbols and 12 sub-carriers per RB. In each RB, 14 REs are used for overhead. At a spectral efficiency of 2 bits / RE, how many RBs are needed to transmit the voice packet.

   (c) If the system has 51 RBs in bandwidth with one slot every 0.5 ms, what is the fraction of RBs used by the VoIP application?

5. *HARQ Errors:* For each of the following events, state what will occur:

   - The PDU can eventually recovered through HARQ, or
   - The PDU cannot be recovered through HARQ and will need to be recovered from a high-layer ARQ protocol (e.g. at the RLC or TCP layer).

   (a) A DL PDCCH for an initial transmission is not seen by the UE, so it does not even know that there is a DL data transmission.

   (b) The UE decodes the DL data and sends an ACK to the gNB. But, the gNB mistakes the ACK for a NACK.

   (c) The UE fails to decode the DL data and sends a NACK to the gNB. But, the gNB mistakes the NACK for an ACK.

6. *Power and SNR estimation:* Suppose that we have two groups of reference symbols:

   - Zero-power RS that contain noise only,

   $$r_k = w_k, \quad w_k \sim \mathcal{CN}(0, N).$$

   On these symbols, we compute a noise estimate,

   $$\widehat{N} = \frac{1}{K} \sum_{k=1}^{K} |r_k|^2,$$

   where K is the number of symbols over which we average.

- Non zero-power RS with signal and noise,

$$r_k = h_k x_k + w_k, \quad h_k \sim \mathcal{CN}(0, E_s), \quad w_k \sim \mathcal{CN}(0, N),$$

and $|x_k| = 1$ is known to the receiver. On these symbols, we compute a signal power estimate

$$\widehat{S} = \frac{1}{M} \sum_{k=1}^{K} |r_k|^2,$$

where M is the number of symbols over which we average.

(a) Show that $\widehat{N}$ and $\widehat{S}$ can be written as a scaled chi-squared distributions with a certain number of degrees of freedom. You can look up this distribution in any source.

(b) Show that the ratio $\widehat{S}/\widehat{N}$ can be written as a scaled F-distribution distribution with a certain number of degrees of freedom. You can look up this distribution in any source.

(c) Suppose we use

$$\widehat{\gamma} = \max\left\{0, \frac{\widehat{S}}{\widehat{N}} - 1\right\}$$

as the estimate of the true SNR $\gamma = E_s/N$. Plot the probability that the SNR is accurate within 0.5 dB as a function of K with $K = M$. You can use the MATLAB function `fcdf`. Assume the true SNR is, $\gamma =$ 3 dB.

7. *CSI estimation bias:* Suppose that in a group of K symbols, reference symbols x_k are received as

$$r_k = h x_k + w_k, \quad w_k \sim \mathcal{CN}(0, N), \quad |h x_k|^2 = E_s,$$

where h is an unknown channel, N is the noise power, and E_s is the received signal energy. We channel and noise estimates via

$$\widehat{h} = \frac{\sum_{k=1}^{K} x_k^* r_k}{\sum_{k=1}^{K} |x_k|^2}, \quad \widehat{N} = \frac{\alpha}{K} \sum_{k=1}^{K} |r_k - \widehat{h} x_k|^2.$$

(a) Find the constant α such that the noise estimate is unbiased. That is,

$$\mathbb{E}\left[\widehat{N}\right] = N.$$

(b) Suppose that you obtain an accurate estimate of the noise $\widehat{N} = N$ (for example, by averaging over large numbers of groups). How would you get an unbiased estimate of E_s?

8. *Rate matching:* Suppose you send 200 bits with a rate 1/2 convolutional code with constraint length $K = 7$.

(a) How many coded bits are output from the convolutional encoder? Remember to include the tail bits.

(b) Suppose you want to send the data on 150 QPSK symbols. How many bits should be punctured or repeated?

9. *Comparing Chase and IR:* Suppose that a TX can create mother codes that, on an AWGN channel, require an SNR γ and provide a rate per symbol of

$$R(\gamma) = \min\{\rho_{\max}, \alpha \log_2(1+\gamma)\},$$

where $\rho_{\max}$ is the maximum spectral efficiency and α is the fraction that the code achieves within the Shannon rate. Now suppose we use this code for HARQ with K transmissions. Suppose that all the symbols in transmission k experience some SNR γ_k, $k = 1, \ldots, K$.

(a) Suppose, we use Chase combining where we create a packet from the mother code and retransmit it in each of the K transmissions. For each target γ, find the condition on $\gamma_1, \ldots, \gamma_K$ that the packet will pass. Also, find the rate, $R_{\text{chase}}(\gamma)$ that will be achieved if the packet passes after K transmissions.

(b) Next, suppose we use IR where we create a longer packet and transmit a fraction $1/K$ symbols in each transmission. For each target γ, find the condition on $\gamma_1, \ldots, \gamma_K$ that the packet will pass. Also, find the rate, $R_{\text{IR}}(\gamma)$ that will be achieved if the packet passes after K transmissions.

(c) Set $K = 3$ and generate random i.i.d. γ_k that are exponentially distributed with an average of 3 dB (i.e. independent Rayleigh fading). Generate $n = 1000$ instances of this channel using MATLAB. By varying the target SNR γ, plot the probability that the packet the packet passes after K transmissions, vs. the rates $R_{\text{chase}}(\gamma)$ and $R_{\text{IR}}(\gamma)$, for both Chase and IR combining.